GAMES I

MATH CIRCLE (INTERMEDIATE) 2/12/2012

In the following problems each player wants to win. Determine who can always achieve this goal (i.e. who has the winning strategy) and why:

1) Two players take turns putting pennies on a round table, without piling one penny on the top of another. the player who cannot place a penny loses.

2) Two players take turns placing bishops on the squares of a chessboard, so that they cannot capture each other (the bishops may be placed on square of any color). The player who cannot move loses.

3) Two players take turns placing rooks on a chessboard so that they cannot capture each other. the loser is the player who cannot place a rook.

4) There are two piles of 7 stones each. At each turn, a player may take as many stones as he chooses, but only from one of the piles. The loser is the player who cannot move.

5) There are three piles of stones: one with 10 stones, one with 15 stones, and one with 20 stones. At each turn, a player can choose on of the piles and divide it into two smaller piles. The loser is the player who cannot do this.

6) There are two piles of stones. One has 30 stones and the other has 20 stones. Players take turns removing as many stones as they please, but from one pile only. The player who cannot remove stone loses.

Challenge 1) Ten 1's and ten 2's are written on a blackboard. In one turn, a player may erase any two figures. If the two figures erased are identical, they are replaced with a 2. If they are different, they are replaced with a 1. The first player wins if a 1 is left at the end, and the second player wins if a 2 is left.

Challenge 2) Two players rake turns breaking a piece of chocolate consisting of 5×10 small squares. At each turn, they may break along the division lines of the squares. The player who first obtain a single square of chocolate wins.

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg "Mathematical Circles (Russian Experience)"
- Previous UCLA Math Circle notes

Warm up 1) The plane is colored using two colors. Prove that there are two identically colored points exactly 1 meter apart.

Warm up 2) Two children take turns breaking up a rectangular chocolate bar 6 squares wide by 8 square long. They may break the bar only along the divisions between the squares. If the bar breaks into several pieces, they keep breaking the pieces up until only the individual squares remain. The player who cannot make a break looses the game. Who will win?