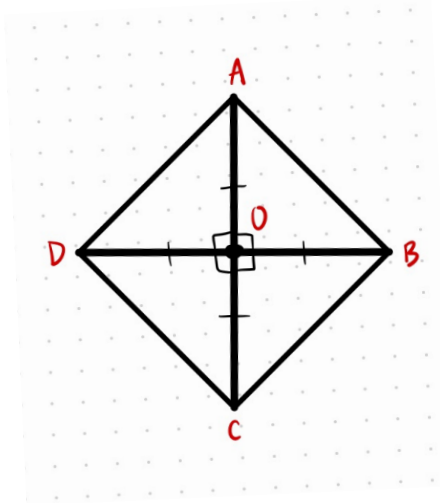


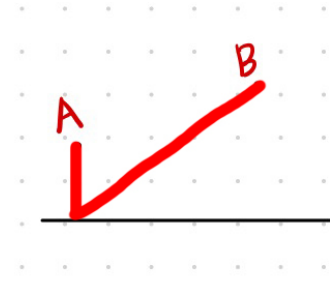
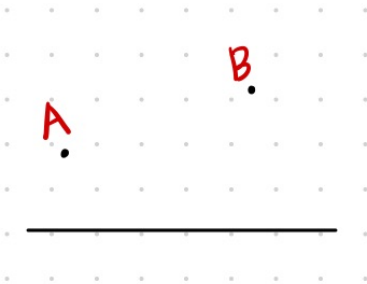
CHALLENGE PROBLEMS: (Attempt on a separate piece of paper)

1.) Notice that the quadrilateral below is such that  $AO$ ,  $BO$ ,  $CO$ , and  $DO$  all have equal length and moreover  $AC$  and  $BD$  are perpendicular (meaning they form right angles). Prove that the quadrilateral must be a square.



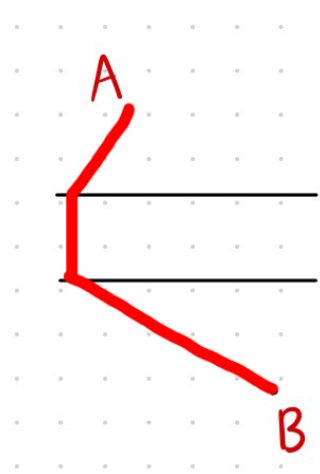
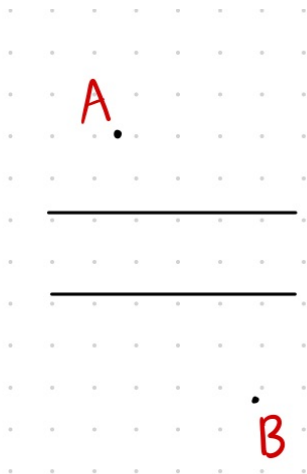
Hint: a square is defined to be a quadrilateral where all four sides have equal length and all four corners form right angles.

2.) What is the shortest path that starts at  $A$ , goes straight to the line, and then goes straight to  $B$  in the diagram below to the left? An example of such a path (not the shortest) is on the right.



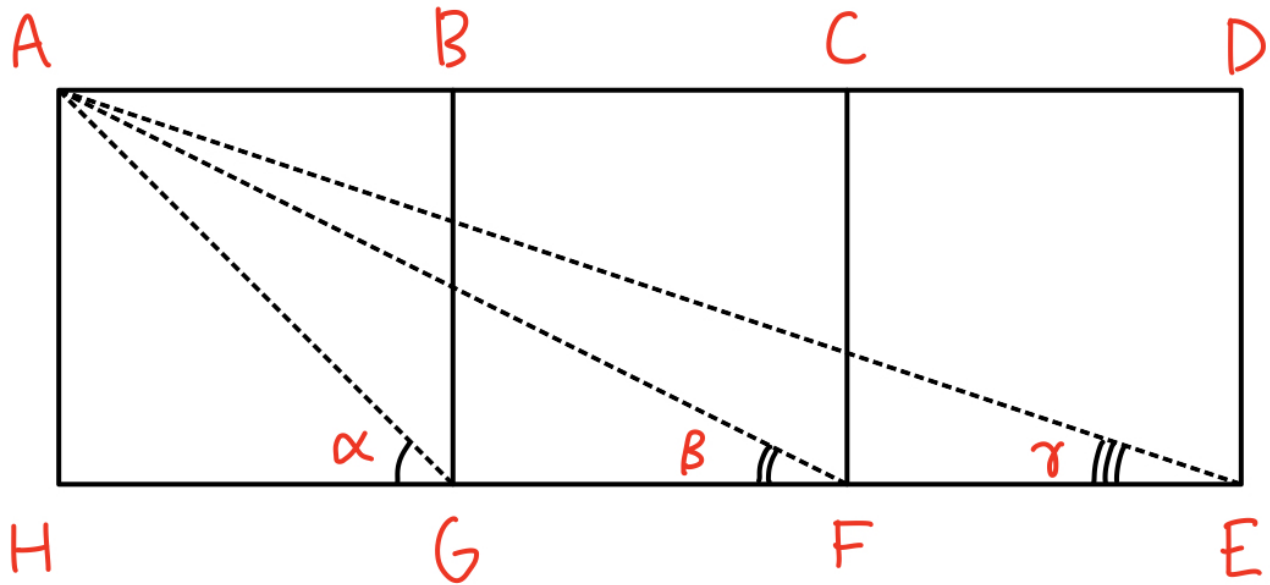
Hint: use triangle congruence.

3.) What is the shortest path that starts at  $A$ , goes straight to the top line, drops down perpendicular to the bottom line, and then goes straight to  $B$  in the diagram below to the left? An example of such a path (not the shortest) is on the right.

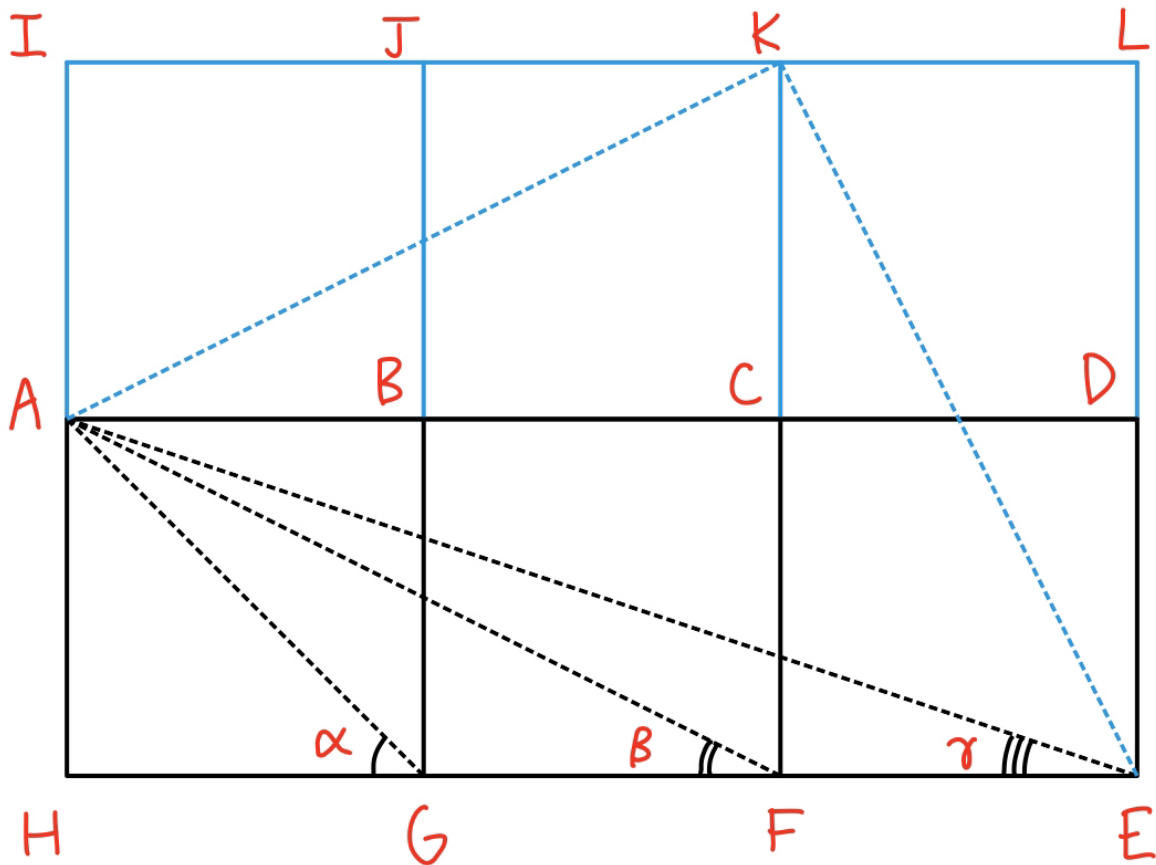


Hint: use triangle congruence.

4.) Below,  $ABGH$ ,  $BCFG$ ,  $CDEF$  are all squares with equal side lengths. Recall our definition of a square is a quadrilateral with equal side lengths and interior angles all being right angles. Prove that the angles  $\alpha$ ,  $\beta$ ,  $\gamma$  are such that  $\alpha^\circ + \beta^\circ + \gamma^\circ = 90^\circ$  (meaning  $\alpha$ ,  $\beta$ ,  $\gamma$  sum up to a right angle).



Hint: this problem is extra difficult. Try thinking of auxiliary constructions that will allow you to use triangle congruence effectively. By auxiliary constructions, I mean try drawing in extra lines or shapes to help.



Hint 2: consider the above auxiliary construction and try to use triangle congruence. Again this is a tough problem so it is okay if you struggle on it.