CHALLENGE PROBLEMS: (Attempt on a separate piece of paper)

1.) Notice that the quadrilateral below is such that $AO$, $BO$, $CO$, and $DO$ all have equal length and moreover $AC$ and $BD$ are perpendicular (meaning they form right angles). Prove that the quadrilateral must be a square.

[Diagram of a square with labeled vertices]

Hint: a square is defined to be a quadrilateral where all four sides have equal length and all four corners form right angles.

2.) What is the shortest path that starts at $A$, goes straight to the line, and then goes straight to $B$ in the diagram below to the left? An example of such a path (not the shortest) is on the right.

[Diagram showing a path from A to B]

Hint: use triangle congruence.

3.) What is the shortest path that starts at $A$, goes straight to the top line, drops down perpendicular to the bottom line, and then goes straight to $B$ in the diagram below to the left? An example of such a path (not the shortest) is on the right.

[Diagram showing a path from A to B]

Hint: use triangle congruence.
4.) Below, \(ABGH, BCFG, CDEF\) are all squares with equal side lengths. Recall our definition of a square is a quadrilateral with equal side lengths and interior angles all being right angles. Prove that the angles \(\alpha, \beta, \gamma\) are such that \(\alpha^\circ + \beta^\circ + \gamma^\circ = 90^\circ\) (meaning \(\alpha, \beta, \gamma\) sum up to a right angle).

Hint: this problem is extra difficult. Try thinking of auxiliary constructions that will allow you to use triangle congruence effectively. By auxiliary constructions, I mean try drawing in extra lines or shapes to help.

Hint 2: consider the above auxiliary construction and try to use triangle congruence. Again this is a tough problem so it is okay if you struggle on it.