PROBLEMS

January 9, 2012

1. Draw the graph of $f(x) = 1 - |x|$ for x on $[-1, 1]$. Now draw a figure that explains $f(x) = 1 - |x|$ without drawing a graph, as you saw done for $f(x) = |x|$. It's easier if you look at $f(x)$ on $[-1, 0]$ and on $[0, 1]$ separately.

2. Find the fixed points of $f(x) = x^3$. Draw the graph and locate the fixed points. What happens if you also draw the graph of $g(x) = x$ on the same figure?

3. Use Bolzano's Intermediate Value Theorem to prove that the equation $\sqrt{x+1} + \sqrt{x+2} = 2$ has a solution on $[-1, 1]$.

4. Describe how $f(z) = z^2$ behaves on the unit disc D. What are its fixed points?

5. The *complex conjugate* of $z = x + yi$ is $\bar{z} = x - yi$. Describe $f(z) = \bar{z}$ on D and find its fixed points.

6. Draw a figure to show how $f(z) = z^3$ behaves on the unit circle C. There are two fixed points; can you find them? How many fixed points does $f(z) = z^4$ have on C. Can you find the pattern for the number of fixed points of $f(z) = z^k$ on C?

7. Describe a smooth map of D such that $f(z) = \bar{z}$ on C that has no fixed points except on C.

8. Define a map of D such that $f(z) = z^3$ on C that has no fixed point points except on C. (Hint: Recall Schirmer's construction for $f(z) = z^2$).