Note that by carrying out the above constructions, we just have (nearly) proven the following very important theorem.

Theorem 1 *Two triangles in the Euclidean plane are congruent if either of the following holds.*

- **SSS** *(Example 1) The sides are pairwise congruent (or the side lengths are pairwise equal).*

$$a \cong a',\ b \cong b',\ c \cong c'$$

- **SAS** *(Problem 12) The triangles have congruent angles and the sides adjacent to the angles are pairwise congruent.*

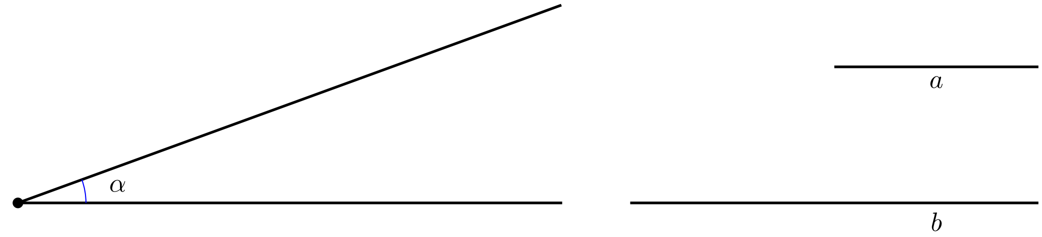
$$b \cong b',\ \alpha \cong \alpha',\ c \cong c'$$

- **ASA** *(Problem 13) The triangles have congruent sides , and the adjacent angles are pairwise congruent.*

$$\alpha \cong \alpha',\ c \cong c', \beta = \beta'$$

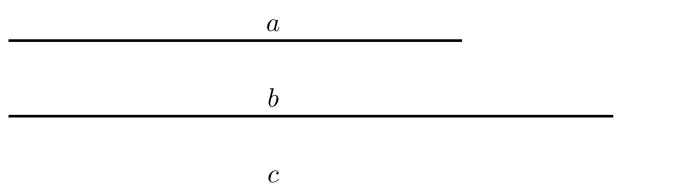
Problem 14 (SSA) *At the top of the next page, construct a triangle with the angle α given below as well as with the side b adjacent to α and with the side a opposite to the angle. How many non-congruent solutions do you get?*

(Try to use only a compass and straight edge! Don't use a protractor to measure the angle. Refer to the lesson 2 packet if you need a reminder)



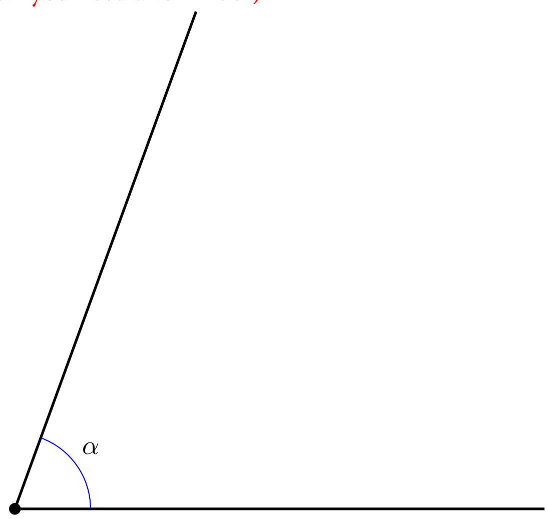
Note 1 *Unlike SSS, SAS, and ASA, SSA can produce non-congruent triangles!*

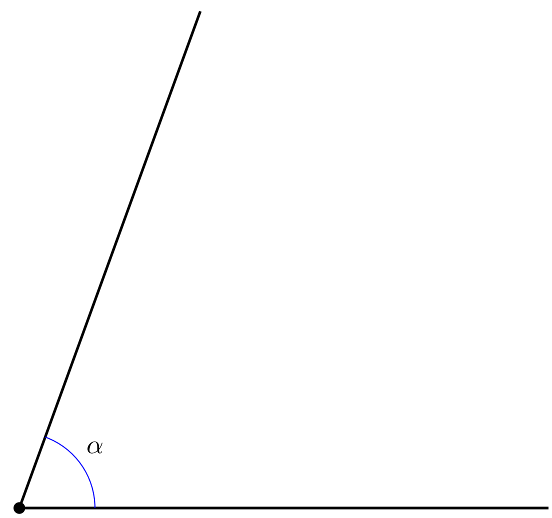
Homework Problem 1 *Use a compass and a ruler to construct a triangle having the following sides.*



Homework Problem 2 *Use a compass and a ruler to construct an angle congruent to the angle α below in two different ways. Use an auxiliary triangle on this page and an auxiliary circumferences on the next one.*

(Try to use only a compass and straight edge! Don't use a protractor to measure the angle. Refer to the lesson 2 packet if you need a reminder)

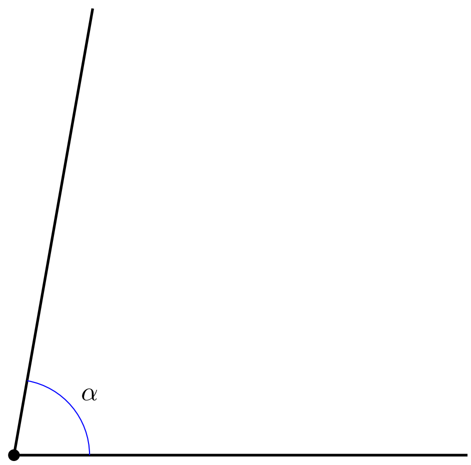


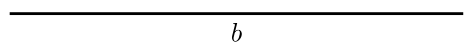


Which method do you like more? Why?

A triangle is called *isosceles*, if it has two sides of equal length.

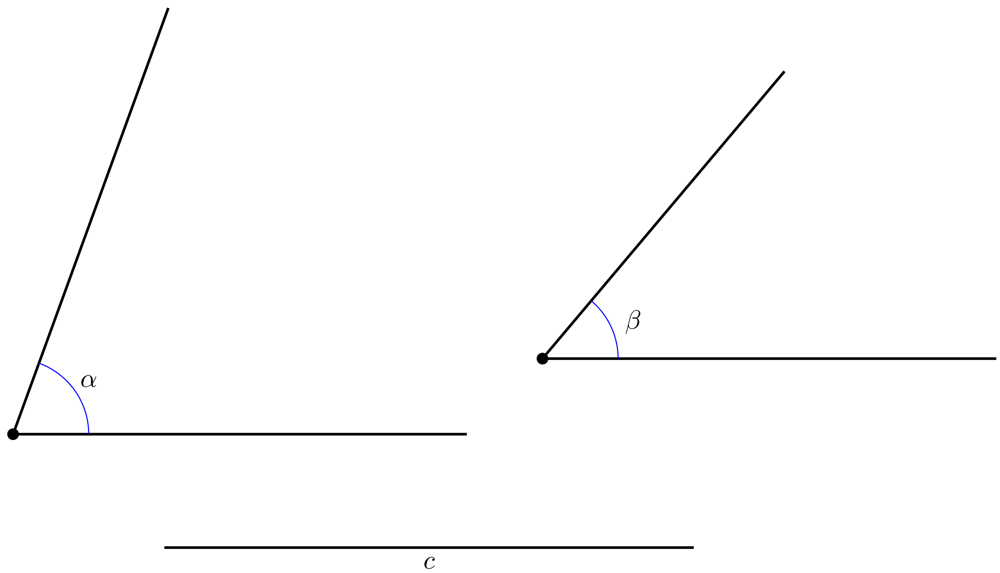
Homework Problem 3 *In the space below, construct an isosceles triangle with the angle α and with the sides adjacent to it congruent to the segment b.*
(Try to use only a compass and straight edge! Don't use a protractor to measure the angle. Refer to the lesson 2 packet if you need a reminder)





Homework Problem 4 *Construct a triangle with the side c and with adjacent angles α and β given below.*

(Try to use only a compass and straight edge! Don't use a protractor to measure the angle. Refer to the lesson 2 packet if you need a reminder)



A triangle is called *equilateral* if all of its sides have equal length.

Homework Problem 5 *In the space below, construct an equilateral triangle with* $2''$ *sides.*

Greek alphabet

You will find the Greek alphabet in the table below.

Letter	$A\ \alpha$	$B\ \beta$	$\Gamma\ \gamma$	$\Delta\ \delta$	$E\ \epsilon$	$Z\ \zeta$	$H\ \eta$	$\Theta\ \theta$
Name	alpha	beta	gamma	delta	epsilon	zeta	eta	theta
Letter	$I\ \iota$	$K\ \kappa$	$\Lambda\ \lambda$	$M\ \mu$	$N\ \nu$	$\Xi\ \xi$	$O\ o$	$\Pi\ \pi$
Name	iota	kappa	lambda	mu	nu	xi	omicron	pi
Letter	$P\ \rho$	$\Sigma\ \sigma$	$T\ \tau$	$Y\ \upsilon$	$\Phi\ \phi$	$X\ \chi$	Ψ	$\Omega\ \omega$
Name	rho	sigma	tau	upsilon	phi	chi	psi	omega

Question 2 *What did the word "alphabet" originate from?*

Question 3 *What is the meaning of the expression "from alpha to omega"?*

Self-test questions

- What is a ray?

- What is an angle?

- What is a straight angle?

- What is an angle complementary to the given one?

- What is an angle supplementary to the given one?

- What geometric figures do we call congruent?

- What is 1°?

- How many degrees are there in a full angle? In a straight angle?

- What angles are called vertical?

- Are vertical angles congruent? Why or why not?

- What is the meaning of the word *polygon*?

- Can congruent triangles have sides of different length?

- How can one construct a triangle with given sides using a compass and straightedge as tools?

- How can one construct an angle congruent to the given one using a compass and straightedge as tools?

- What is the meaning of the word *adjacent*?

- How can one construct a triangle with a compass and straightedge, given its angle and two adjacent sides?

- How can one construct a triangle with a compass and straightedge, given its two angles and the side adjacent to both of them?

- Formulate the **SSS, SAS, ASA** theorem.

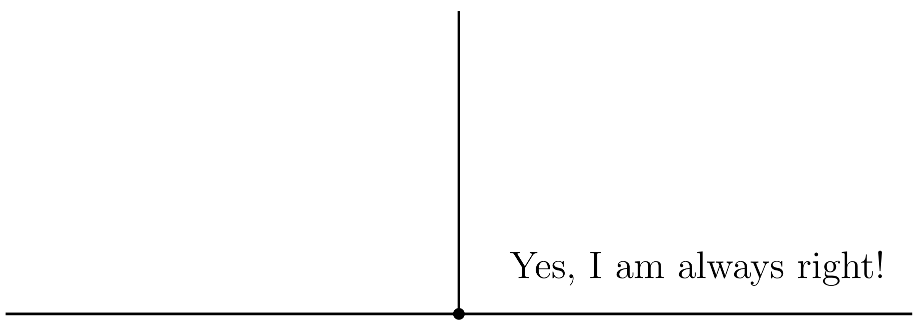
- Does the **SSA** construction always produce a triangle congruent to the given one? Why or why not?

Oleg Gleizer LAMC

Introduction to Geometry Lesson 3

The right angle

An angle is called *right* if it is congruent to its supplementary angle.



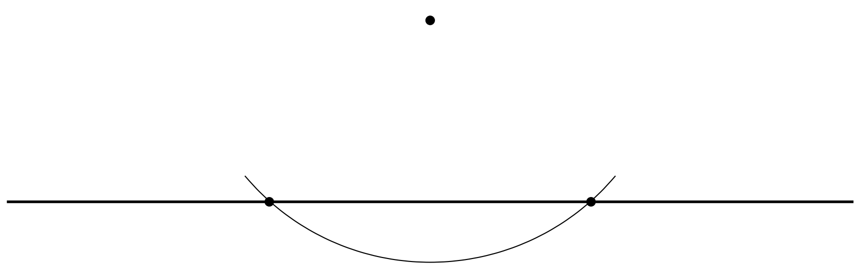
An angle smaller than a right angle is called *acute*. An angle larger than a right angle, but smaller than a straight angle is called *obtuse*.

Problem 1 *Use degrees to write an algebraic statement showing that the angle* α *is*

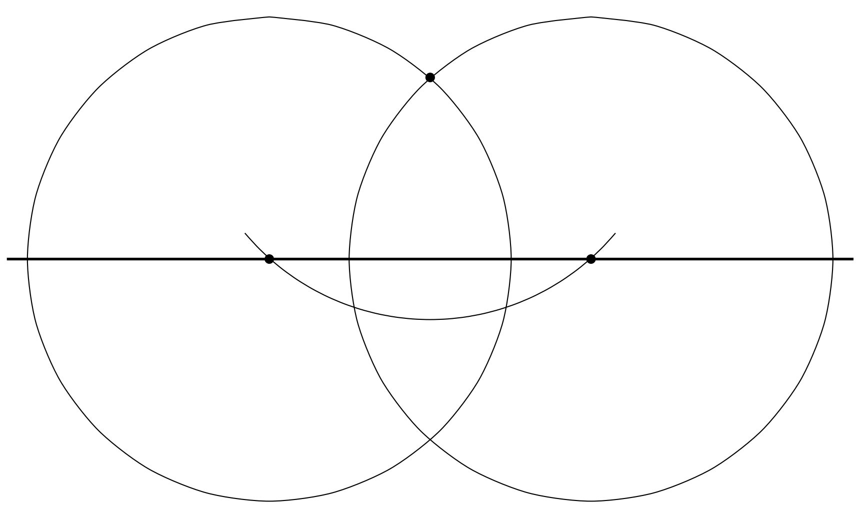
- *right:*
- *acute:*
- *obtuse:*

Given a straight line and a point not lying on the line, the following procedure enables one to construct the right angle such that one of its sides is a part of the line and the other passes through the point, using a compass and straightedge as tools.

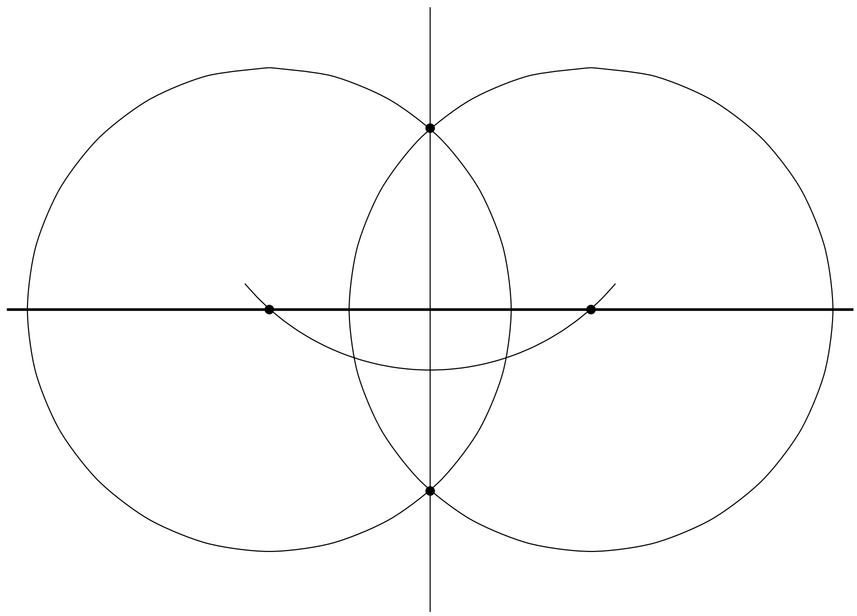
Step 1: spread the legs of the compass wide enough so that the circumference centered at the given point meets the line at two distinct points. Mark the points.



Step 2: keeping the radius the same, draw the circumferences centered at the two points on the line.

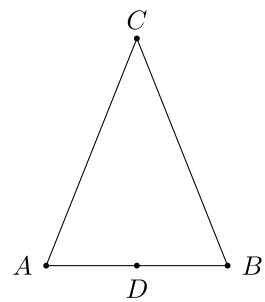


Step 3: mark the point opposite to the original one and draw a straight line through the two points.



The following sequence of problems explains why the above method works and explores some of the features of the involved geometric objects.

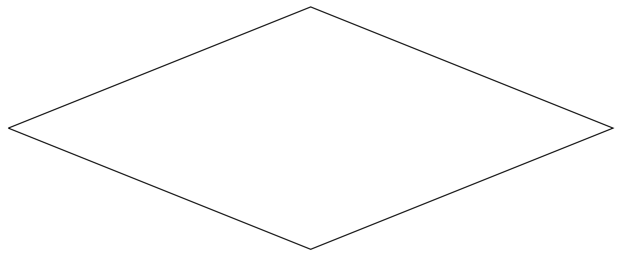
Recall that a triangle is called *isosceles* if it has two sides of equal length. Consider the triangle ABC such that $AC = BC$. Let D be the midpoint of the side AB.



Problem 2 *Prove that the angle ADC is right.*
(Try to use triangle congruence! Try not to use that a right angle is 90 degrees, but rather use the definition which says a right angle is congruent to its supplementary angle.)

Problem 3 *Prove that the angles of an isosceles triangle opposite to the congruent sides are congruent.*
(Try to use triangle congruence!)

A quadrilateral is called a *rhombus* if all its sides have the same length.



Problem 4 *Prove that opposite angles of a rhombus are congruent.* (Try to use triangle congruence!)

Problem 5 *Prove that diagonals split angles of a rhombus in halves.* (Try to use triangle congruence!)

Problem 6 *Prove that diagonals of a rhombus intersect at a right angle.*
(Try to use triangle congruence! Try not to use that a right angle is 90 degrees, but rather use the definition which says a right angle is congruent to its supplementary angle.)

Question 1 *Does Problem 6 explain why the method of constructing a right angle presented at the beginning of the lesson works? Why or why not?*

Problem 7 *Prove that diagonals of a rhombus split each other in halves.* (Try to use triangle congruence!)

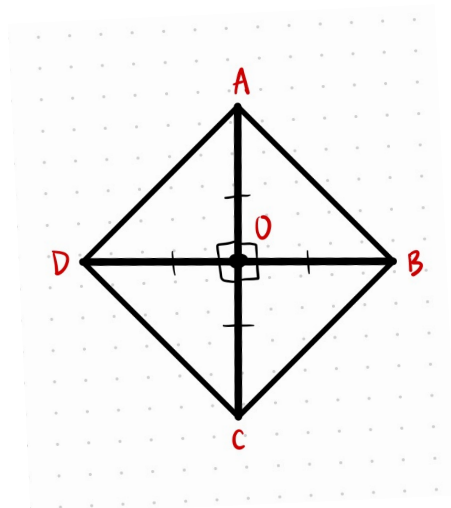
Problem 8 *Use a compass and a straightedge to find the midpoint of the following segment.* (Do not measure with a ruler.)

A quadrilateral with all the four sides of equal length and all the four angles right is called a *square*.

Problem 9 *Use a compass and a ruler to construct a square with* $2''$ *side lengths in the space below.*

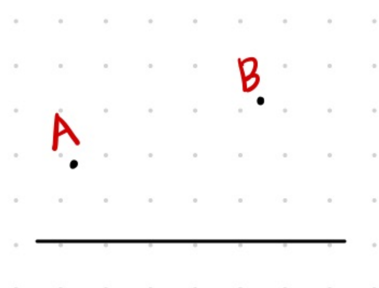
CHALLENGE PROBLEMS: (Attempt on a separate piece of paper)

1.) Notice that the quadrilateral below is such that *AO, BO, CO*, and *DO* all have equal length and moreover *AC* and *BD* are perpendicular (meaning they form right angles). Prove that the quadrilateral must be a square.



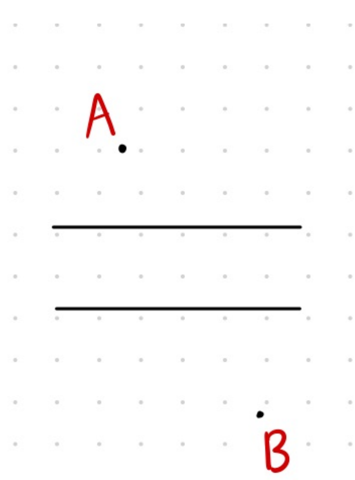
Hint: a square is defined to be a quadrilateral where all four sides have equal length and all four corners form right angles.

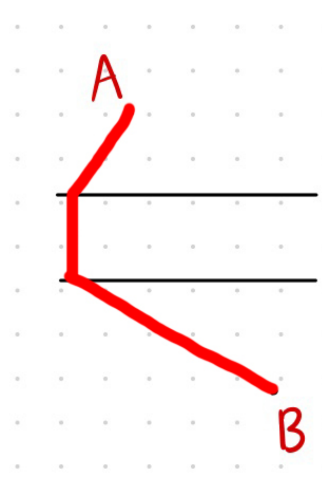
2.) What is the shortest path that starts at *A*, goes straight to the line, and then goes straight to *B* in the diagram below to the left? An example of such a path (not the shortest) is on the right.



Hint: use triangle congruence.

3.) What is the shortest path that starts at *A*, goes straight to the top line, drops down perpendicular to the bottom line, and then goes straight to *B* in the diagram below to the left? An example of such a path (not the shortest) is on the right.





Hint: use triangle congruence.