Cryptarithmetic, also know as cryptarithm, alphametics, or word addition, is a math game of figuring out unknown numbers represented by words. Different letters correspond to different digits. Same letters correspond to same digits. The first digit of a number cannot be zero.

**Problem 1** Solve the following cryptarithm.

\[
\begin{array}{c}
H \ E \\
+ \ H \ E \\
\hline
S \ H \ E \\
\end{array}
\]
The following cryptarithm\textsuperscript{1} is a bit more complicated.

\[
\begin{array}{cccc}
S & E & N & D \\
+ & M & O & R & E \\
\hline
M & O & N & E & Y \\
\end{array}
\]

Let’s try to solve it together. First, \(S + M \geq 10\) because the result carries over from the fourth column to the fifth. A sum of single digit numbers cannot exceed 18, so \(M = 1\).

\[
\begin{array}{cccc}
S & E & N & D \\
+ & 1 & O & R & E \\
\hline
1 & O & N & E & Y \\
\end{array}
\]

Now, \(S + 1 \geq 10\). This can only happen if either \(S = 9\) or \(S = 8\). Let us consider the latter case. If \(S = 8\), then \(O = 0\) and one is carried over from the third column, implying \(E = 9\) and \(N = 0\). But, since \(O = 0\), \(N \neq 0\). Thus, \(S = 9\).

\[
\begin{array}{cccc}
9 & E & N & D \\
+ & 1 & O & R & E \\
\hline
1 & O & N & E & Y \\
\end{array}
\]

\textsuperscript{1}Invented by Henry Dudeney, published in Strand Magazine in 1924.
Looking at the fourth column, we see that $O$ can be either zero or one. In the latter case, one is carried over from the third column, so $E + 1 \geq 9$. $E \neq 9$ because $S = 9$, so $E = 8$. If this is the case, then one is carried over from the second column and $N = 0$. In the second column, this gives $0 + R = 10 + E = 18$, so $R = 18$ and is a single digit number at the same time. Thus, $O = 0$ and $E = N - 1$.

\[
\begin{array}{cccc}
9 & N-1 & N & D \\
+ & 1 & 0 & R & N-1 \\
\hline
1 & 0 & N & N-1 & Y
\end{array}
\]

Suppose that one is not carried over from the first column to the second. Then $N + R = 10 + N - 1$ implying $R = 9$, an impossibility since $S = 9$. Thus, one is carried over from the first column and $N + R + 1 = 10 + N - 1$, giving us $R = 8$ and $D + N - 1 \geq 10$.

\[
\begin{array}{cccc}
9 & N-1 & N & D \\
+ & 1 & 0 & 8 & N-1 \\
\hline
1 & 0 & N & N-1 & Y
\end{array}
\]

Since $Y \neq 0, 1$, the latter inequality can be strengthened to $D + N > 12$. The single-digit number $N$ can only take values 3, 4, 5, 6, 7. Let us consider these possibilities case by case.
$N = 3$ implies $D > 9$.

$N = 4$ implies $D > 8$, but $D \neq 9$.

$N = 5$ implies $D > 7$, but $D \neq 8, 9$.

$N = 7$ implies $D > 5$, so $D = 6$ and $N - 1 = 6$.

Finally, $N = 6$ implies $D > 6$, so $D = 7$. $7 + 5 = 12$, so $Y = 2$. Here comes the solution.

\[
\begin{array}{c}
9 & 5 & 6 & 7 \\
+ & 1 & 0 & 8 & 5 \\
\hline
1 & 0 & 6 & 5 & 2 \\
\end{array}
\]
Problem 2 Solve the following cryptarithm.

\[
\begin{array}{cccc}
F & O & R & T & Y \\
+ & T & E & N \\
\hline
T & E & N \\
\hline
S & I & X & T & Y
\end{array}
\]
Problem 3 Solve the following cryptarithm.

\[
\begin{array}{cccccccc}
M & O & T & H & E & R \\
+ & F & A & T & H & E & R \\
\hline
P & A & R & E & N & T \\
\end{array}
\]
Problem 4 Solve the following cryptarithm.

\[
\begin{align*}
\text{O} & \quad \text{N} & \quad \text{E} \\
+ & \quad \text{T} & \quad \text{H} & \quad \text{R} & \quad \text{E} & \quad \text{E} \\
\hline
\text{F} & \quad \text{O} & \quad \text{U} & \quad \text{R} \\
\hline
\text{E} & \quad \text{I} & \quad \text{G} & \quad \text{H} & \quad \text{T}
\end{align*}
\]