Week 3: Hard similar triangles

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1 Old problems

New problems are hard. We recommend you first to complete the previous worksheet.

2 New problems

Problem 1.
The vertices of parallelogram $A_1B_1C_1D_1$ lie on the sides of parallelogram $ABCD$ (point A_1 lies on side AB, point B_1 lies on side BC, etc.). Prove that the centers of both parallelograms coincide.

Problem 2.
Point P is taken on the bisector of the angle. The straight line passing through point P cuts segments of length a and b on the sides of the angle. Prove that the value of $\frac{1}{a} + \frac{1}{b}$ does not depend on the choice of this line.

Problem 3.
Through point P of the median CC_1 of triangle ABC, lines $AA1$ and $BB1$ are drawn (points A_1 and B_1 lie on sides BC and CA). Prove that $A_1B_1 \parallel AB$.

3 Secret problems

Problem 4.
a) Points A, B, and C lie on one straight line; points A_1, B_1, and C_1 lie on another straight line. Prove that if $AB_1 \parallel BA1$ and $AC_1 \parallel CA_1$, then $BC_1 \parallel CB_1$.

b) Points A, B, and C lie on one straight line and points A_1, B_1, and C_1 are such that $AB_1 \parallel BA1$ and $AC_1 \parallel CA_1$ and $BC_1 \parallel CB_1$. Prove that A_1, B_1 and C_1 lie on one line

Problem 5.
In triangle ABC bisectors AA_1 and BB_1 are drawn. Prove that the distance from any point M of A_1B_1 to line AB is equal to the sum of distances from M to AC and BC.