

THE TRIANGLE INEQUALITY I

MATH CIRCLE (INTERMEDIATE) 1/29/2012

1) Side AC of triangle ABC has length 3.8, and side AB has length 0.6. If the length of side BC is an integer, what is this length?

2) Prove that the length of any side of a triangle is not more than half its perimeter.

3) The distance from Leningrad to Moscow is 660 kilometers. From Leningrad to the town of Likovo it is 310 kilometers, from Likovo to Klin it is 200 kilometers, and from Klin to Moscow is 150 kilometers. How far is it from Likovo to Moscow?

4) Point O is given on the plane of square $ABCD$. Prove that distance from O to one of the vertices of the square is not greater than the sum of the distances from O to the other three vertices.

5) Prove that the sum of the diagonals of a convex quadrilateral is less than the perimeter but more than half the perimeter.

6) Prove that the sum of the diagonals of a convex pentagon is greater than the perimeter but less than double the perimeter.

Challenge 1) A woodsman's hut is in the interior of a peninsula which has the form of an acute angle. The woodsman must leave his hut, walk to one shore of the peninsula, then the other shore, then return home. How should he choose the shortest such path?

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg "Mathematical Circles (Russian Experience)"
- Previous UCLA Math Circle notes

Warm up 1) A king wants to build 6 fortresses and connect each pair of them by a road. Draw a scheme of fortresses and roads such that there are only 3 crossroads, each formed by 2 intersecting roads.

Warm up 2) Prove that for any three points A, B, C we have $AC > |AB - BC|$.