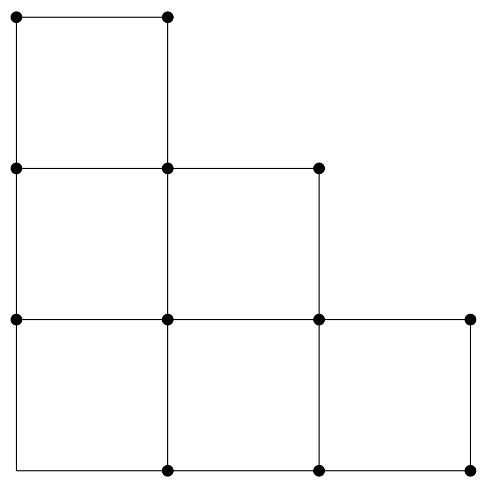
LAMC Junior Circle 3/17/2013

Math Wrangle

Problem 1 *Remove two matches so that there are left four squares with a one-match-long side.*



Problem 2 *How many times 5% of 5 is less than 50% of 50?*

Problem 3 *Solve the following cryptarithm.*

$$\begin{array}{rccccc} & F & O & R & T & Y \\ + & & & T & E & N \\ & & & T & E & N \\ \hline & S & I & X & T & Y \end{array}$$

Problem 4 *How many six-digit numbers do there exist such that each of them has at least one even digit?*

Problem 5 *Can an intersection of a triangle and a quadrilateral be an octagon?*

Problem 6 *Insert either a plus or a minus between the numbers below so that the result equals 20.*

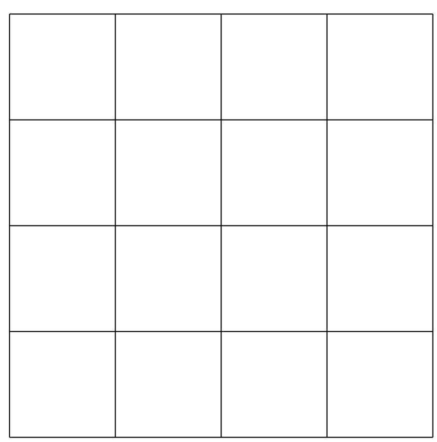
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

15 17 18 19 20 = 20

Problem 7 – *Captains' fight.*

Without using a calculator, find the square root of the number 12345678987654321.

Problem 8 *A square is made of 16 smaller squares of equal size.*



Shade some of the smaller squares so that every shaded square has a common side with three non-shaded squares and each non-shaded square has a common side with one shaded square.

Problem 9 *For a positive integer* x*, let* $s(x)$ *be the sum of its digits. For example,* $s(35) = 8$*.*

a. Solve the equation $x + s(x) = 2013$*.*

b. Has the equation $x + s(x) + s(s(x)) = 2014$ *any solutions? Why or why not?*

Problem 10 *One day Winnie-the-Pooh woke up and noticed that his old grandfather's clock didn't move. He wound up the clock and went to see Rabbit who had a modern atomic clock running on batteries that needed no winding and always showed precise time. After eating most of Rabbit's honey (but not all, so he didn't get stuck this time), Winnie-the-Pooh got back home and set up his own clock to show the correct time. How did Pooh Bear do that?*

Problem 11 *Nicholas with his son and Peter with his son went fishing. Nicholas caught as many fish as his son did. Peter caught three times as many fish as his son. The total number of the fish caught was 25. How many fish did Nicholas catch?*

Problem 12 *Consider the following 300-digit number*

$$112112\ldots112,$$

the number 112 *written consecutively 100 times. How many different 298-digit numbers can we get by crossing out two digits of the original number?*

Problem 13 *Is it possible to make the following product*

$$1! \times 2! \times \ldots \times 99! \times 100!$$

a perfect square by marking out one factorial? Why or why not?

Problem 14 *Two people are playing the following game. Given a round table and a supply of coins, all of the same size, they take turns placing coins on the table. The coins are not allowed to touch one another. A player wins when her/his opponent cannot place a coin on the table. Find the winning strategy for the game.*

Problem 15 *Prove that*

$$1 + 3 + \ldots + (2n - 1) = n^2$$

for $n = 1, 2, 3, \ldots$.

Problem 16 *What is the next number in the following sequence?*

1, 11, 21, 1112, 3112, 211213, 312213, 212223, 114213, ...