Week 2: Similar triangles

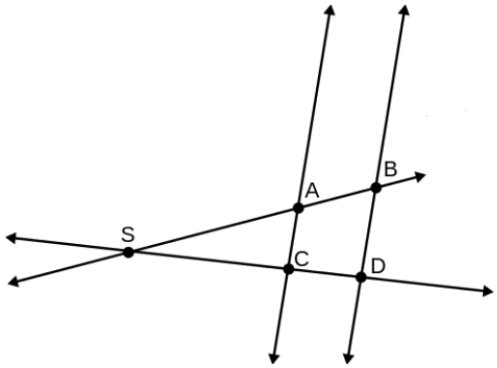
Nikita

1 Main information

Definition 1.
Triangle ABC is called ***similar*** to triangle $A_1B_1C_1$ (denoted by $ABC \sim A_1B_1C_1$) if and only if one of the following equivalent conditions is satisfied:

SSS) $AB : BC : CA = A_1B_1 : B_1C_1 : C_1A_1$;
SAS) $AB : BC = A_1B_1 : B_1C_1$ and $\angle ABC = \angle A_1B_1C_1$;
AA) $\angle ABC = \angle A_1B_1C_1$ and $\angle BAC = \angle B_1A_1C_1$.

Theorem 1 (Thales or intercept theorem)**.**
Suppose S is the intersection point of two lines and A, B are the intersections of the first line with the two parallels, such that B is further away from S than A, and similarly C, D are the intersections of the second line with the two parallels such that D is further away from S than C. Then $SAC \sim SBD$.



2 Problems

Definition 2.
A *middle line* of a triangle is the segment that connects the midpoints of two sides.

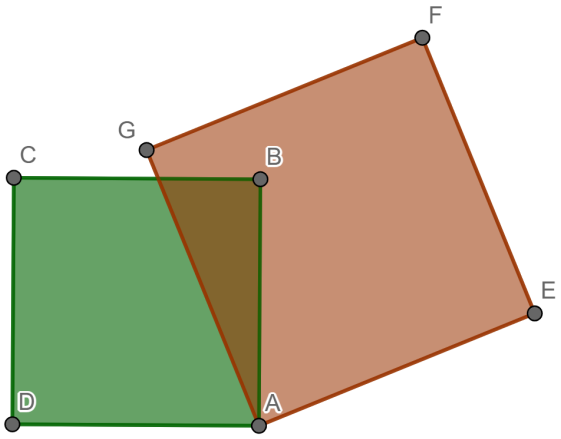
Problem 1. (a) Draw a triangle, then draw all three middle lines of the triangle.

(b) Prove that a middle line is parallel to the third side and is equal to half its length.

Problem 2.
In an acute-angled triangle ABC the altitudes AA_1 and BB_1 are drawn. Prove that $A_1C \cdot BC = B_1C \cdot AC$.

Problem 3.
Squares $ABCD$ and $AEFG$ share a vertex A. Prove that **a)** $ABE = ADG$;

b) $ACF \sim ABE$.



Problem 4.
Prove that the medians of a triangle intersect at one point and are divided by this point in a ratio of $2:1$, counting from the vertex.

Problem 5.
Using ruler and compass, divide a given segment in a ratio of $5:7$.

Problem 6.
The square $PQRS$ is inscribed in the triangle ABC so that the vertices P and Q lie on the sides AB and AC, and the vertices R and S lie on the side BC. Express the length of the side of the square in terms of side a and the height h_a.

Problem 7.
Prove that the midpoints of the sides of an arbitrary quadrilateral are the vertices of a parallelogram. For which quadrilaterals is this parallelogram a rectangle, for which is it a rhombus, for which is it a square?