Week 1: Warming up

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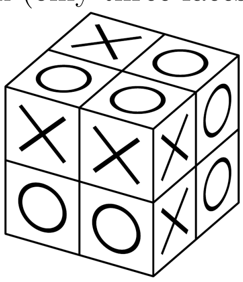
Problem 1.
Let f be a quadratic polynomial with integer coefficients (i.e. $f(x) = ax^2 + bx + c$, where a, b and c are integer). It is known that $f(0)$ is even and $f(1)$ is odd. What can be said about $f(2)$?

Problem 2.
What is the maximal number of rooks one can place on a chessboard so that no two of them attack each other?

Problem 3.
What is the maximal number of bishops one can place on a chessboard so that no two of them attack each other?

Problem 4.
a) SpongeBob split each face of the $b \times 2 \times 2$ cube into unit squares and ordered Patrick to write crosses in some squares, and zeros in the rest so that each square bordered on the sides with two crosses and two zeroes. The figure shows how Patrick completed the task (only three faces are visible). Prove that Patrick was wrong.

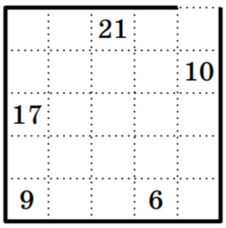


b) Help Patrick to complete the task correctly. It is enough to describe at least one correct arrangement.

Problem 5.
Solve the cryptarithm $NO + GUN + NO = HUNT$.

Problem 6.
A maze was drawn on checkered paper: a 5×5 square (outer wall) with a one-cell-wide exit, as well as inner walls running along the grid lines. In the picture, we have hidden all the inner walls from you.

Draw how they could be located, knowing that the numbers in the cells indicate the least number of steps in which it was possible to leave the maze, starting from this cell. A step is made to the cell adjacent to the side, if they are not separated by a wall. One example is enough.



Problem 7.

There are 6 different apples on the table. Tanya put them 3 of them on a balance scale against other 3, and the scales remained in balance. Sasha laid out the same apples in a different way: 2 apples on one part and 4 on another, and the scale remained in balance again. Prove that you can put one apple on one part of the scale and two on the other so that the scale remains in balance.