

GRAPH II

MATH CIRCLE (INTERMEDIATE) 1/22/2012

1) There are 10 students in a class. Is it possible that each has exactly 3 friends in the class?

2) In Smallville there are 15 telephones. Can these be connected so that each telephone is connected with exactly 7 others?

3) In Smallville there are 15 telephones. Can these be connected so that there are 4 telephones, each connected to 3 others, 8 telephones each connected to 6 others, and 3 telephones, each connected to 5 others?

4) A king has 19 vassals. Can it happen that each vassal has either 1, 5, or 9 neighbors?

5) John, coming home from Disneyland, said that he saw there an enchanted lake with 7 islands, to each of which there led either 1, 3, or 5 bridges. Is it true that at least one of these bridges must lead to the shore of the lake?

6) Prove that a graph with n vertices, each of which has degree at least $(n - 1)/2$, is connected.

Challenge 1) In a certain country, 100 roads lead out of each city, and one can travel along those roads from any city to any other. One road is closed for repairs. Prove that one can still get any city to any other.

Challenge 2) A piece of wire is 120 cm long. Can one use it to form the edges of a cube, each of whose edges is 10 cm?

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg “Mathematical Circles (Russian Experience)”
- Previous UCLA Math Circle notes

Warm up 1) Is it possible to label the edges of a cube using the numbers 1 through 12 in such a way that the sums of the numbers on any two faces of the cube are equal?

Warm up 2) Can a kingdom in which 3 roads lead out of each city have exactly 100 roads?