

GRAPH I

MATH CIRCLE (INTERMEDIATE) 1/15/2012

1) We have n houses in a village. Every house is connected to another house by a road. What is the total number of roads in this village?

2) In Smallville there are 15 telephones. Can they be connected by wires so that each telephone is connected with exactly five others?

3) In a certain kingdom, there are 100 cities, and four roads lead out of each city. How many roads are there altogether in the kingdom?

4) There are 30 students in a class. Can it happen that 9 of them have 3 friends each (in the class), eleven have 4 friends each, and ten have 5 friends each?

5) There are 30 students in a class. Can it happen that 20 of them have 19 friends each (in the class), and 10 of them have 9 friends each?

6) Prove that the number of people who have ever lived on earth, and who have shaken hands an odd number of times in their lives, is even.

Challenge 1) In the country of Figura there are nine cities, the the names 1, 2, 3, 4, 5, 6, 7, 8, 9. A traveler finds that two cities are connected by an airplane route if and only if the two-digit number formed by naming one city, then the other, is divisible by 3. Can the traveler get from City 1 to City 9?

Challenge 2) Can 9 line segments be drawn in the plane, each of which intersects exactly 3 others?

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg “Mathematical Circles (Russian Experience)”
- Previous UCLA Math Circle notes

Warm up 1) Is it possible to write numbers 1 through 100 in a row in such a way that the (positive) difference between any two neighboring numbers is not less than 50?

Warm up 2) A chessboard has the form of a cross, obtained from 4×4 chessboard by deleting the corner squares. Can a knight travel around this board, pass through each square exactly once, and end on the same square he starts on?