Spring Quarter Game

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1 Easy Problems

1. Find positive integers x, y, z such that $28x + 30y + 31z = 365$.

   Answer: $1, 4, 7$ ($2, 1, 9$ is also acceptable, but $0, 7, 5$ – give them another try since 0 is not positive).

2. Steph came up with a positive integer. Klay multiplied it by 5 or by 6. Draymond added 5 or 6 to the result. Kevin subtracted 5 or 6 from the result. In the end, the number was equal to 73. What number did Steph have originally? List all possibilities.

   Answer: 12.

3. How many 9-digit numbers have an even sum of digits?

   Answer: $45 \cdot 10^7$.

4. Daisy used to live in a square room with an integer side length. Now she's moved into an apartment with two rooms, but the same total area. One of the rooms in her new apartment is $7m^2$, and the other is a square room with integer side length. What is the total are of Daisy's apartment?

   Answer: $16m^2$.

5. All page numbers of a book have 492 digits in total. How many pages are there in the book, is they are numbered starting with 1?

   Answer: 200.

6. 37 numbers are written in a row, so that the sum of any 6 consecutive numbers is 29. If the first number is 5, what could the last number be? List all possibilities.

   Answer: 5.

2 Medium Problems

1. 50 apples and oranges are lined up on the table. Next to every apple there is an orange. What is the biggest possible number of apples?

   Answer: 33.

2. Iris thought of 4 numbers, and calculated their pairwise sums. Five of them turned out to be $70, 110, 120, 180$ and 230. What is the value of the sixth sum?

   Answer: 190.

3. All sides of a triangle are distinct integers. What is its smallest possible perimeter?

   Answer: 9.

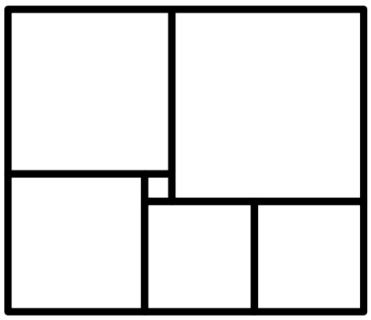
4. A safe is locked with a code consisting of digits from 1 to 6 in some order (each digit appears once). We know that the code is even and in every pair of neighbouring digits one is divisible by another. What is the code?

   Answer: 513624.

5. When 200 candies are distributed in a class, there are guaranteed to be two people with the same number of candies (perhaps 0). What is the smallest possible number of students in the class?

   Answer: 21.

6. The side of the smallest square in the picture is 1 cm. What is the side of the biggest square?



   Answer: 7.

3 Hard Problems

1. Every person in a group of 12 is either a knight who always tells the truth or a liar who always lies. The first person said: "The number of liars among us is divisible by 1". The second one said "The number of liars among us is divisible by 2" and so on, until the last one said: "The number of liars among us is divisible by 12". How many knights could there be in the group?

   Answer: $3, 4$ or 12 (all should be listed, if not all listed, give them the second try).

2. Kyle has cards with digits $1, 2, 3, 4$ on them – two cards with each digit. He want to make an 8-digit number out of them in such a way that there is exactly 1 digit between the 1's, 2 digits between the 2's, 3 digits between the 3's and 4 digits between the 4's. Give an example of such number.

   Answer: 41312432 (others are possible).

3. Water, milk, juice and lemonade are stored in a cup, a jar, a glass and a pitcher. It is known that neither milk nor water are in the cup; lemonade is between the pitcher and juice; neither water nor lemonade are in a jar; the glass is next to the jar and to milk. What is stored where?

   Answer: lemonade – cup, water – glass, milk – pitcher, juice – jar.

4. Find angles of a triangle if each of them is a square of an integer (in degrees).

   Answer: $16°, 64°, 100°$

5. Make a magic square out of numbers 1 through 9: position them in a 3×3 square so that the sum of numbers in each row, column and diagonal is the same.

   Answer (unique up to rotations and symmetry, be careful):

2	7	6
9	5	1
4	3	8

6. Find the smallest positive integer with the product of digits equal to 2016.
   Answer: 4789.

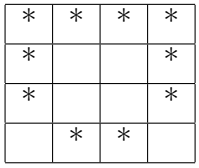
4 Really Hard Problems

1. In an isosceles triangle ABC with $AB = BC$ the angle bisector BD is two time shorter than the angle bisector AE. Find the angles of $\triangle ABC$.

   Answer: $36°, 36°, 108°$.

2. What is the biggest possible number of bishops that can be positioned on a 4×4 chessboard so that every bishop attacks an even number of other bishops? Draw an example.

   Answer (example is not unique): 10.



3. Replace the asterisks with numbers to obtain a correct column multiplication example.

$$\begin{array}{cccccc} & & & * & * & * \\ \times & & & * & 8 & * \\ \hline & & * & * & * & * \\ & & * & * & * & \\ * & * & * & * & & \\ \hline * & * & * & * & * & 9 \end{array}$$

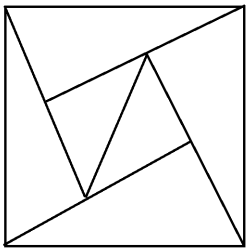
   Answer: $121 \times 989 = 119669$

4. Digits from 9 to 1 are written on the board. It is allowed to insert pluses between some of them. What is the largest possible 3-digit sum which can be achieved by doing that?

   Answer (if they just guess 999 without providing this decomposition – that's fine): $999 = 9 + 8 + 7 + 654 + 321$.

5. Cut a square into triangles so that every triangle has a shared part of a side with exactly 3 other triangles (just shared vertex does NOT count as shared part of a side).

   Answer (not unique):



6. Movers Ethan and James are moving some boxes. Ethan takes 1 minute to move a small box (including time to get back), and James takes 3 minutes. However, Ethan takes 6 minutes to move a big box, and James takes 5. In total they have to move 10 small and 10 big boxes. What is the smallest amount of time they can take? The movers cannot carry more than 1 box at a time, and cannot carry a box together.

   Answer: 33.