

## 1. SEQUENCES

**Problem 1. (10 points)** A hiker begins an ascent of Mount Whitney on Saturday morning, reaching the summit by nightfall. She then spends the night at the summit, and starts down the mountain the following morning, reaching the bottom on Sunday nightfall. Prove that, at some precise time of day, they was exactly at the same altitude on Tuesday as he was on Monday.

**Problem 2. (5 points)** Determine if  $d(x, y) = |x^2 - y^2|$  is a metric on  $\mathbb{R}$ . If not, which property of a metric is not satisfied?

**Problem 3. (10 points)** Give an example of a continuous function for which the Intermediate Value Theorem does not hold. (Hint: Pay attention to the domain of the function.)

**Problem 4. (10 points)** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be the function  $f(x) = x^3 - x + 100$ . Find an interval of length at most 1 on which  $f$  has a zero.

**Problem 5. (5 points)** A man was born on the island Crete off the coast of Greece. One day while shopping for Terra Chips<sup>TM</sup> at the “local” supermarket, he introduces himself to the shop owner whose name happens to be Richard. What do you call this situation?