Herd Immunity
Math Circle
May 23, 2021

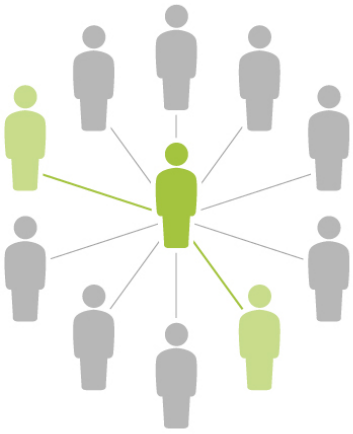
1. What is herd immunity?

enough ppl immune in a population such that virus runs out of places to spread

2. Define R_0 to be the number of people, on average, who will be infected by a single infected person. Note that R_0 is pronounced "R naught."

If an infected person encounters 10 other people,
a virus with $R_0 = 2$ would infect 2 of them, on average.

10 people encountered


2 people become infected

3. Consider an illness where people who are infected later recover, and they are at that point immune to future infections. Over time, how would R_0 change for this ailment?

R_0 gets smaller

less ppl get infected over time

4. Suppose that a vaccine offers complete immunity. If 5 out of every 10 people take the vaccine, how would that change R_0 in the above example? $R_0 = 2$

original R_0 → $\frac{2}{10} \times \frac{5}{10} = \frac{10}{100} = \frac{1}{10}$ ← R_0 (only 5 ppl can get sick)

$R_0 = 1$

5. Consider an ailment with $R_0 > 4$. Describe what will happen if the community has no vaccines and cannot otherwise protect itself.

community keeps getting sick :(exponentially

6. Now consider an ailment with $R_0 < 1$. What happens this time?

illness spreads slower over time, but never stops

7. Herd immunity takes hold when enough people are immune such that the effective value of R_0 is below 1. Can you calculate the required percentage?

Define **N** to be the average number of people with whom an infected person comes in contact. In the prior examples, N was 10.

N = 10

Define **V** to be the number of vaccinated people in each group of N.

Define **R** to be the number of people in each group of N people who would be infected but for the vaccine.

What value of **V** will cause the effective rate of infection to drop to 1?

N − V

V = ?
N = 10
R = 2

$\frac{R \to 2}{N \to 10} \cdot \frac{?}{10} = \frac{1}{10} \leftarrow N$ ← herd immunity!

ppl who would get sick originally

ppl who can get sick

V = 5

*with variables

$$\frac{R}{N} \cdot \frac{N-V}{N} = \frac{1}{N}$$

$$R \cdot \frac{(N-V)}{N} = 1$$

$$\frac{N-V}{N} = \frac{1}{R}$$

$$N-V = \frac{N}{R}$$

$$N - \frac{N}{R} = V$$

$$V = N\left(1 - \frac{1}{R}\right)$$

8. Should we expect the herd immunity threshold to vary from community to community? Why or why not? Give examples.

9. Suppose that a community has reached herd immunity. What factors might cause the relevant ailment to start increasing despite the supposed immunity?

10. If we wanted to calculate the real-world herd immunity threshold for Covid-19 in Los Angeles, what data would we need?

11. News accounts report that the Pfizer Covid-19 vaccine is 95% effective. What does that mean exactly? Does it mean that, out of every 100 people given the vaccine, we should expect that 5 will fall ill?

12. Test your understanding using the real numbers. Pfizer recruited 43,661 volunteers for its first large vaccine trial. Half were given the real vaccine and half were given a placebo. After six weeks, 170 had fallen ill with Covid. Out of those 170, 8 had actually received the vaccine. Is this data consistent with the claim that the vaccine is 95% effective?

13. Last week, the New York Yankees baseball team announced nine cases of Covid-19 in fully vaccinated members of the team organization. Specifically, one player, three coaches, and five staff members tested positive. Assuming that everyone in the Yankees organization was vaccinated with a vaccine that is said to be 75% effective, is this size of a "breakthrough" surprising? Convince me.