

Genetics and Probability

Olga Radko Endowed Math Circle | Chynna Swift | May 2021

Disclaimer: many of the biological topics in this worksheet are simplified.

Part I: Important Terms and Concepts

Genetics is the study of inherited characteristics. Statistics and probability are used frequently in the study of genetics.

Genes are passed from parent to offspring. Genes determine our traits. You have 2 copies of each gene: one from your biological mother, and one from your biological father.

Alleles are variant forms of a gene.

An example of gene versus allele:

Gene: eye color

Alleles: blue eyes, green eyes, brown eyes....

We have 2 alleles for each gene: one from mom and one from dad. What determines *which* trait we physically express?

A Dominant alleles: only one dominant allele copy is needed for it to be expressed physically. We denote dominant alleles with a capital letter (for example, A)

a Recessive alleles: two allele copies are needed for the trait to be expressed physically. We denote recessive alleles with a lowercase letter (for example, a)

Let's look at an example of dominant and recessive alleles for hair color. Given each person's **genotype**, tell me their hair color. Remember, each person has 2 alleles for each gene.

Example 1) Gene: hair color
Brunette allele: A
Blonde allele: a

Bob: AA hair color: brunette

Cindy: Aa hair color: brunette

Dylan: aa hair color: blonde

When you have two copies of the same allele, (AA or aa) you are **homozygous** for that gene. When you have 2 different alleles (Aa), you are **heterozygous** for that gene.

Dedicated to Olga Radko, for believing in me and welcoming me with open arms into the math circle. We're carrying on your legacy every day.

Example 2) A couple (mom Aa, dad Aa) are having a baby. What is the probability that the baby will have blonde hair?

*Remember, the baby receives one allele from each parent, so what are the possible combinations? Do a Punnett square! A Punnett square tells us the possible outcomes for the couple's kids.

gives you all outcomes in correct ratios!

which combos give blonde hair?

	A	a
A	AA	Aa
a	Aa	aa

$$\frac{1}{4}$$

- Now you try! What's the probability that a blonde mom and a heterozygous brunette dad will have a kid with brown hair? (Hint: first find the **genotype** of the parents)

	A	a
a	Aa	aa
a	Aa	aa

$$\frac{1}{2}$$

- What's the probability that 2 homozygous brunette parents will have a blonde child? A brunette child? Can you figure this out without a Punnett Square?

↑
one

yes

	A	A
a	Aa	Aa
a	Aa	Aa

zero
no "a" to give, so no blondes

Part II: Gender Reveal

- What's the probability of a couple having a daughter as their first-born child?

$$\frac{1}{2}$$

- A couple has 4 daughters. What's the probability that their fifth child will be a son?

still $\frac{1}{2}$

5. A couple has 5 kids. What's the probability of them having 2 boys first, then 3 girls?

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \left(\frac{1}{2}\right)^5 = \boxed{\frac{1}{32}}$$

B B G G G

6. A couple has 5 kids. What's the probability of them having 2 boys and 3 girls? How does this problem differ from problem 5? *order doesn't matter*

BBGGG bbggg ggbbg (1/2)⁵ × 10 = 10/32 = 5/16

GGGbb bggbg gbbgg **must account for birth order!*

bgggb gbbgg gbbgg

5C2 = 5C3

Part 3: What Will They Look Like?

7. B: big ears

b: little ears

Two heterozygous parents have a child with **big ears**. What is the probability that the child is also heterozygous for the big ear gene?

parents: Bb, Bb

	B	b	
B	BB	Bb	$\boxed{\frac{2}{3}}$
b	Bb	bb	

8. A: innie belly button

a: outtie belly button

B: bushy eyebrows

b: non-bushy eyebrows

Sally is heterozygous for the belly button gene and homozygous recessive for the eyebrow-bushiness gene. Tom is also heterozygous for the belly button gene, but he is also heterozygous for the eyebrow-bushiness gene.

- a. What is the probability that their child will have an outtie belly button?

Sally: Aa bb
Tom: Aa Bb

	A	a	
A	AA	Aa	$\boxed{\frac{1}{4}}$
a	Aa	aa	

- b. What is the probability that their child will have bushy eyebrows?

	B	b	
B	BB	Bb	$\boxed{\frac{1}{2}}$
b	Bb	bb	

- c. What is the probability that their child will have an outtie belly button and bushy eyebrows?

$$\frac{1}{4} \times \frac{1}{2} = \boxed{\frac{1}{8}}$$

3

d. Does your answer to part c differ from parts a and b? Why or why not?

yes. child must have both traits

9. B: hairy arms

b: hairless arms

A couple, both heterozygous for hairy arms, have 5 children. What's the probability that at least 1 kid will have non-hairy arms? Explain two ways that you could approach this problem.

method 1

$Bb \times Bb$

	B	b
B	BB	Bb
b	Bb	bb

method 2

All hairy arms = $\left(\frac{3}{4}\right)^5 = \frac{243}{1024} \rightarrow 1 - \frac{243}{1024} = \frac{781}{1024}$

1 non-hairy = $\left(\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4}\right) \times 5$ (bc order matters! is it the first born? middle child?) $\frac{81 \times 5}{1024} = \frac{405}{1024}$

2 non-hairy = $\left(\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}\right) \times 10 = \frac{270}{1024}$

3 non-hairy = $\left(\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}\right) \times 10 = \frac{90}{1024}$

4 non-hairy = $\left(\frac{3}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}\right) \times 5 = \frac{15}{1024}$

5 non-hairy = $\left(\frac{1}{4}\right)^5 = \frac{1}{1024}$

add = $\frac{781}{1024}$

10. Parents are heterozygous for three traits (parents are both Aa, Bb, and Cc).

a. What's the probability of their child being dominant for two traits and recessive for one trait?

	A	a
A	AA	Aa
a	Aa	aa

	B	b
B	BB	Bb
b	Bb	bb

	C	c
C	Cc	Cc
c	Cc	cc

$\left(\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4}\right)^3 = \frac{27}{64}$

b. What's the probability that the child is dominant for 1 trait and recessive for 2 traits?

$\left(\frac{3}{4} \times \frac{1}{4} \times \frac{1}{4}\right)^3 = \frac{9}{64}$

11. A ~~couple~~ ^{couple} has 3 children. What's the probability that, at most, 2 kids have the recessive (aa) trait for freckles given the following genotypes of the parents?

a. If mom is Aa and dad is AA ¹
no kids will have freckles, so rule is satisfied

b. If mom is aa and dad is aa
all kids have freckles, so rule is broken

c. If mom is Aa and dad is Aa

	A	a
A	AA	Aa
a	Aa	aa

all freckles = $\left(\frac{1}{4}\right)^3 = \frac{1}{64} \rightarrow 1 - \frac{1}{64} = \frac{63}{64}$

none freckles = $\left(\frac{3}{4}\right)^3 = \frac{27}{64}$

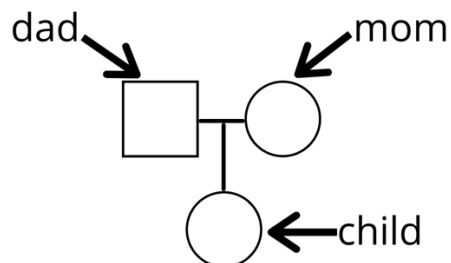
1 freckles: $\left(\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4}\right)^3 = \frac{27}{64}$

2 freckles: $\left(\frac{3}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}\right)^3 = \frac{9}{64}$

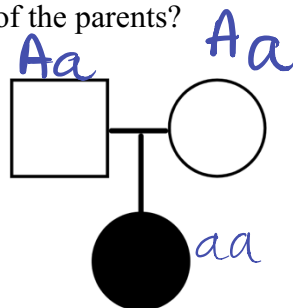
add = $\frac{63}{64}$

Part 4: Ancestry and Pedigrees

Pedigrees give us information about the family's genetic history. Circles represent females, squares represent males. For this worksheet, black circles/squares mean that the individual is homozygous recessive (aa).

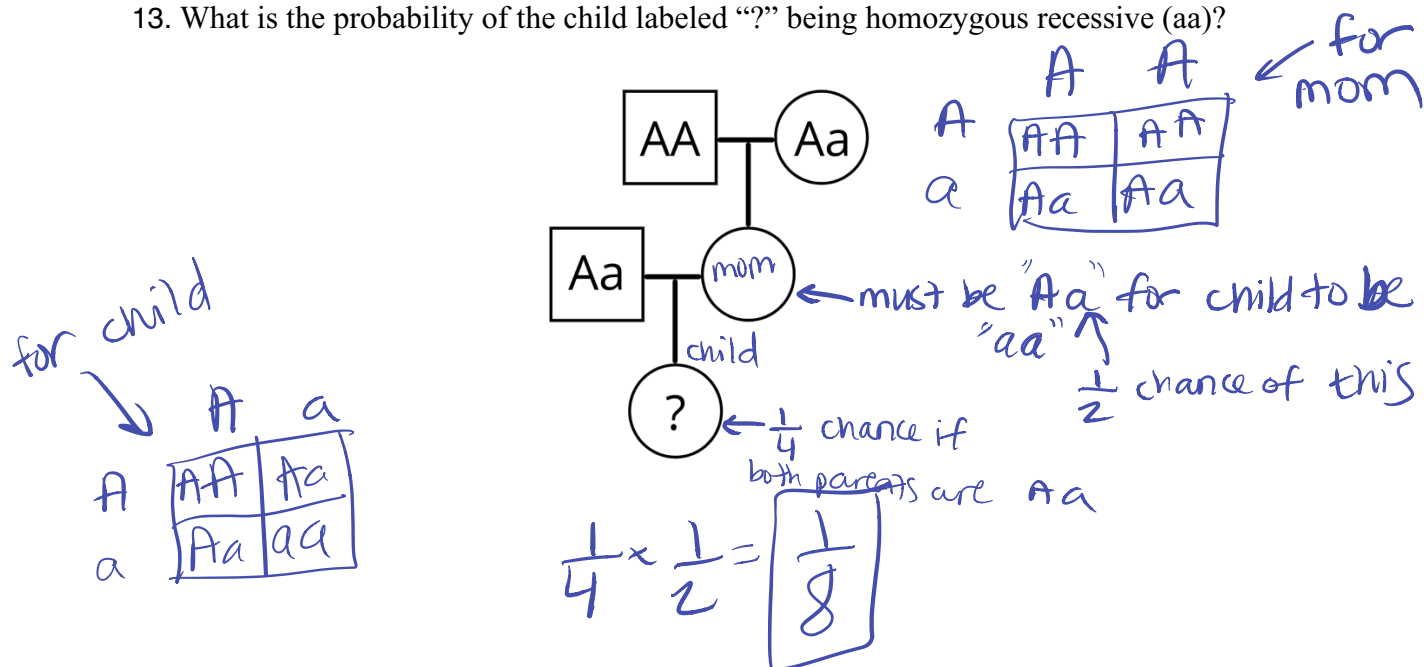


12. Can you figure out the genotypes of the parents?



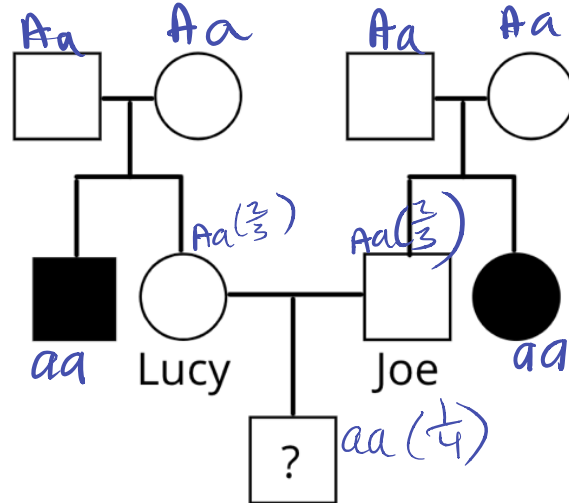
both need "a" to pass to child, but neither parent is aa, so must be Aa

13. What is the probability of the child labeled "?" being homozygous recessive (aa)?



14. Lucy and Joe have siblings with anemia, which is a recessive disease (aa). What is the probability that their child will also have anemia?

* for child to have anemia, both L+J need to be Aa.
 $\frac{2}{3}$ prob of Aa for L+J



	A	a
A	AA	Aa
a	Aa	aa

$$\frac{2}{3} \times \frac{2}{3} \times \frac{1}{4} = \frac{4}{36} = \frac{1}{9}$$

15. If Lucy and Joe's first child has anemia, what is the probability that their second child also has anemia? Is it a different value? Why ~~are~~ why not?

yes, different. Since first child is aa, we know both L+J are Aa.

$$\frac{1}{4}$$

Part 5: Bloody Types

The genotypes for blood types look a little different than the genotypes we are used to writing, but it's the same idea. Here's a list of the 4 blood types and their corresponding allele combinations.

Type A blood: AA or Ai
Type B blood: BB or Bi

Type AB blood: AB
Type O blood: ii

16. If two parents have genotypes Ai and Bi, what are the possible blood types of their children?

	A	i
B	AB	Bi
i	Ai	ii

A, B, AB, O
 all of them!

17. Casey is blood type O, but his parents are blood type A and B. How does that work? What are his parents' genotypes?

Casey = ii

Parents both must have "i" allele to pass down to Casey. So parents are Ai + Bi

Handwritten notes and diagrams illustrating a genetic cross:

Parents: A (square) and B (circle) are crossed.

Genotypes: A is Ai and B is Bi .

Punnett Square:

AB	Bi
Ai	ii

Offspring: The cross results in four possible genotypes: AB , Bi , Ai , and ii .

Phenotypes: The offspring are categorized by blood type: AB (square), Bi (circle), Ai (square), and ii (circle, labeled "type O").

Probability: The probability of an offspring being type O (ii) is $\frac{1}{4}$.

what's weird abt this pedigree? inbreeding!

