

PROPOSITIONAL LOGIC

The language of propositional logic consists of:

propositional variables: P, Q, R, \dots

connective symbols $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

and punctuation: $(,)$.

0) Write down the meaning of the connective symbols:

- \neg :

- \wedge :

- \vee :

- \rightarrow :

- \leftrightarrow :

Well-formed formulas, also called wffs, are defined so that:

- Every propositional variable is a wff.
- If θ and λ are wffs, then so are

$$(\neg\theta), (\theta \wedge \lambda), (\theta \vee \lambda), (\theta \rightarrow \lambda), (\theta \leftrightarrow \lambda)$$

For example,

$$P, (P \rightarrow Q), ((P \vee Q) \rightarrow R), ((\neg P) \wedge (Q \leftrightarrow (\neg R)))$$

are all wffs, while NONE of

$$\rightarrow P, PQ \rightarrow R, (P \vee \wedge R), P \vee Q \rightarrow R$$

are wffs.

2) Translate the following wffs into statements in English.

- P

- Q

- R

- $(P \wedge Q) \rightarrow \neg R$

- $P \leftrightarrow (Q \vee R)$

- $P \rightarrow (Q \rightarrow P)$

- $(\neg Q \vee P) \rightarrow R$

3) Complete the following table:

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$P \leftrightarrow Q$
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T T

T F

F T

F F

4) Suppose we know that P is T, Q is F, and R is T. (Note that this is called a *truth assignment* for P, Q, R .) What is the truth value of the following?

- $(P \wedge Q) \rightarrow R$
- $(\neg P \vee Q) \wedge (R \rightarrow Q)$
- $(Q \rightarrow (P \rightarrow R)) \leftrightarrow ((\neg Q \wedge P) \rightarrow R)$
- $[(\neg P \vee Q) \wedge (R \rightarrow Q)] \vee [(Q \rightarrow (P \rightarrow R)) \leftrightarrow ((Q \wedge P) \rightarrow R)]$

- John is big and dumb, and if John is big then he is not dumb.

- John is big and dumb, if and only if John is not big or not dumb.

6) An advertisement for a tennis magazine says: “If I’m not playing tennis, I’m watching tennis. And if I’m not watching tennis, then I’m reading about tennis.” We can assume that the speaker can only do one of the above activities at a time. By translating the above into propositional logic, figure out what the speaker is doing.

Some problems are taken from:

- H. Enderton “A Mathematical Introduction to Logic”
- S. Russel, P. Norvig “Artificial Intelligence: A Modern Approach”