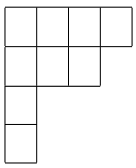
Lesson 6: Young Diagrams

Konstantin Miagkov

Definition 1.
The *partition number* $p(n)$ for a positive integer n is the number of partitions of n into positive parts where partitions different only in ordering of the summands are not distinguished. So for example $9 = 1 + 3 + 1 + 4$ and $9 = 4 + 3 + 1 + 1$ correspond to the same partition.

Definition 2.
The *Young diagram* is a finite collection of boxes, or cells, arranged in left-justified rows, with the row lengths in non-increasing order.
Listing the number of boxes in each row gives a partition of a non-negative integer n, the total number of boxes of the diagram. That gives a one-to-one correspondence between partitions and Young diagrams.
For example the following diagram corresponds to the partition $9 = 4 + 3 + 1 + 1$:



Problem 1.
Compute $p(n)$ for all n from 1 to 8.

Problem 2.
You need to pack cookies into boxes. There are 10 boxes each of which can contain at most 3 cookies. How many ways are there to put 22 cookies into boxes (leaving no box empty)? The boxes are indistinguishable.

Problem 3.
Show that the number of partitions of n into at most k parts each of which is at most ℓ is equal to the number of partitions of n into at most ℓ parts each of which is at most k.

Problem 4.
Show that the number of partitions of n into k parts is equal to the number of partitions of $n + \binom{k}{2}$ into k *distinct* parts.

Problem 5.
Show that the number of partitions of n into distinct odd parts is equal to the number of partitions of n such that their Young diagrams are symmetric with respect to the diagonal.