

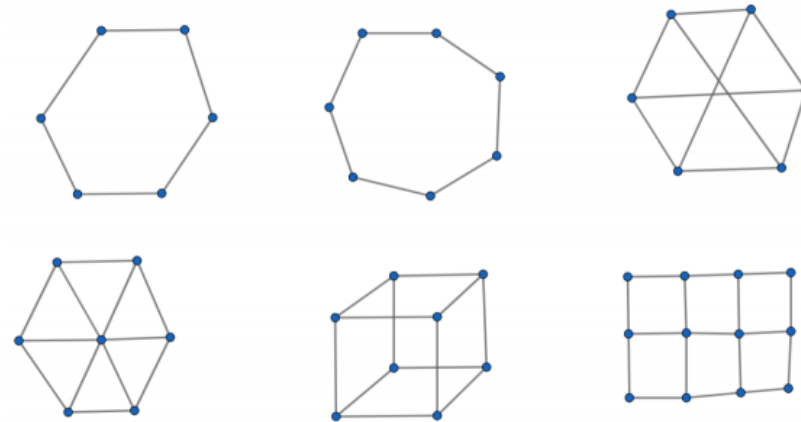
# Graph Theory II

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1. Show that the number of states in the US with an odd number of neighboring states is even.

2. Are the following six graphs bipartite?





- (b) The teacher wants to make every student present one problem they solved on the board. Show that it is possible to choose the problem each student presents so that every problem on the test gets presented exactly once.
6. Is it possible to walk around the  $7 \times 7$  chess board with a knight visiting every square exactly once and finishing back at the starting square?
7. (a) A bipartite graph has  $b$  white and  $r$  black vertices. What is the maximum possible number of edges in this graph?

(b) What is the maximum possible number of edges in a bipartite graph with  $2n$  vertices?

(c) What about  $2n + 1$  vertices?

8. Show that if a graph has no cycles, then it is bipartite.