Sets and Venn Diagrams Part 2
UCLA Olga Radko Math Circle Beginners 2
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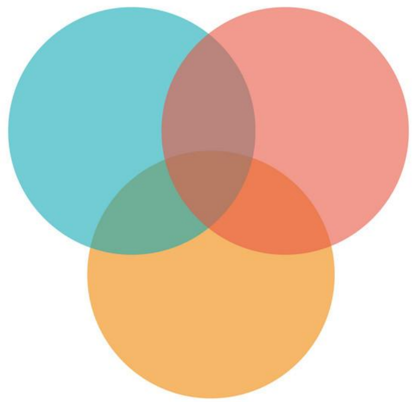


Warm-up:

Consider the following sets:

$$S_1 = \{ 1, 3, 5, \text{cat}, \text{dog}, \text{lion}, \text{pancake}\}$$
$$S_2 = \{ 4, 5, \text{waffles}, \text{dog}, 2, 1\}$$

a. Can you make a new set that contains elements that are found both in A and B?

$S_3=\{1, 5, \text{dog}\}$

b. Can you make a new set that contains elements that are found in either A or B?

$S_4=\{1, 3, 5, \text{cat}, \text{dog}, \text{lion}, \text{pancake}, 4, \text{waffles}, 2\}$

<u>Problem 1:</u> Union and Intersection

a. The set of the elements that belong to the sets A **and** B is called the **intersection** of A and B and is denoted as **A ∩ B**.

   i. *Using the warm-up problem, how can we denote our answer for a?*

      $A \cap B = \{1, 5, \text{dog}\}$

   ii. Let us rewrite this definition completely in the math language.

$$A \cap B = \{x : x \in A \text{ and } x \in B\} \qquad (1)$$

      In this mathematical sentence, the colon reads as *such that*. Translating back into English, *the intersection of the sets A and B is defined as the set of the elements x such that x is an element of A and x is an element of B.*

b. The following is the definition of the **union** of two sets, written down in the math language.

$$A \cup B = \{x : x \in A \text{ or } x \in B\} \qquad (2)$$

   i. Translate definition (2) into English.

      The union of the sets A and B is defined as the set of the elements x such that x is an element of A or x is an element of B.

   ii. Going back to our warmup problem, how would you denote the answer for b?

      $A \cup B = \{1, 3, 5, \text{cat}, \text{dog}, \text{lion}, \text{pancake}, 4, \text{waffles}, 2\}$

c. What is $A \cap \varnothing$ for any set A?

   $A \cap \varnothing = \varnothing$

d. What is $A \cup \varnothing$ for any set A?

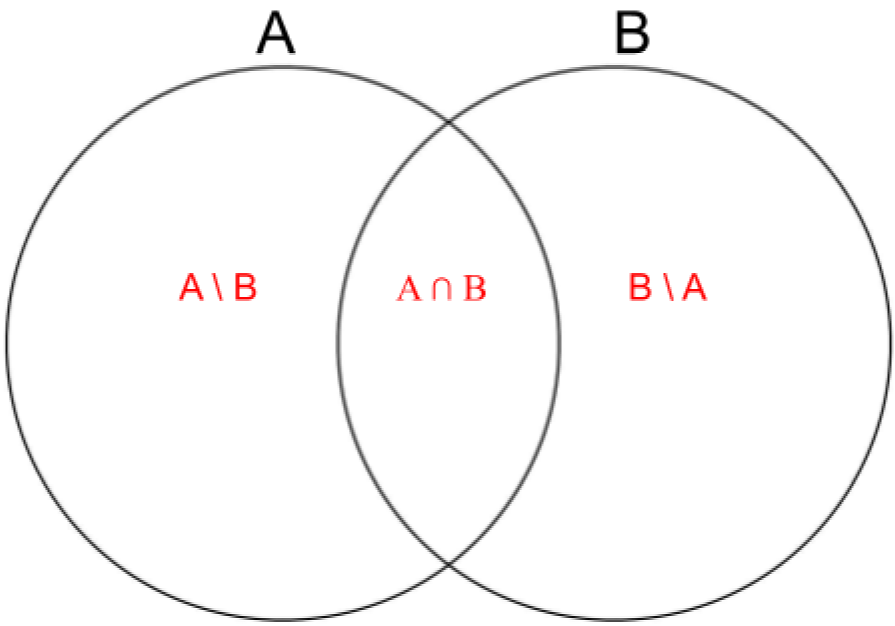
   $A \cup \varnothing = A$

e. Give an example of two sets and of their union different from the ones used so far.

Example: A={1, 2, 3, 4}, B = {1, 3, 5}

$A \cap B = \{1, 3\}$ and $A \cup B = \{1, 2, 3, 4, 5\}$

Problem 2: **Venn Diagrams**



a. Suppose we have two sets A and B. The *difference of the sets **A and B***, the set **A \ B**, is the *set of all the elements of the set A that* **do not belong** *to the set B.*

  i. *Label A \ B in the appropriate section of the Venn Diagram.* See above

  ii. *Translate B / A to English.*

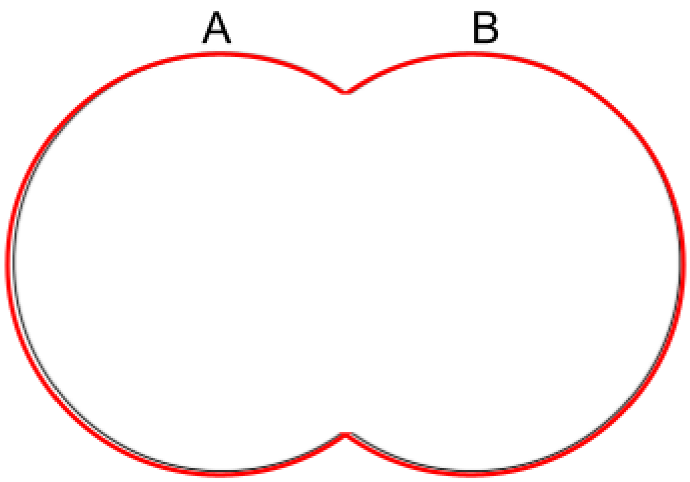
  B / A is the set of all elements of the set B that do not belong to the set A

  iii. *Label B \ A in the appropriate section of the Venn Diagram.* See above

  iv. *Using the notation we have learned so far, how would you label the middle of the Venn Diagram?*

The middle is the intersection of the sets A and B, $A \cap B$

v. *Show the set $A \cup B$ on the Venn Diagram.*



$A \cup B$ is every element in either circle

b. *Let A be the set of spectators at a basketball game. Let B be the set of all the people at the game, spectators, coaches, staff, etc., wearing caps. Describe in your own words the set A \ B.*

A \ B is the set of all spectators at the game not wearing caps.

c. *[Challenge] Use the symbol ∉ to write the definition of the set A \ B in the math language. (Hint: Thinking about how we defined the definition for the union and intersection of two sets)*

$A \setminus B = \{x : x \in A \text{ and } x \notin B\}$

Red Hot Chilli Pepper Problem

a. *How many integers in the set S = {1, 2, 3, . . . , 98, 99, 100} are not divisible by 3?*

67. For a slightly different set, S_1={1, 2, …, 99}, 1/3 of the integers are divisible by 3 (i.e 1 and 2 not are divisible by 3, but 3 is divisible by 3; 4 and 5 are not divisible but 6 is; …; 97 and 98 are not but 99 is), so there are 66 such integers in S_1. We also know that 100 is not divisible by 3, so there are a total of 67 integers that are not divisible by 3 in S.

b. *What is a set?*

Recall that we defined a set as a clearly defined collection of distinct objects, but noted that this was not a very clear or precise definition.

<u>Problem 4:</u> Disjoint Sets

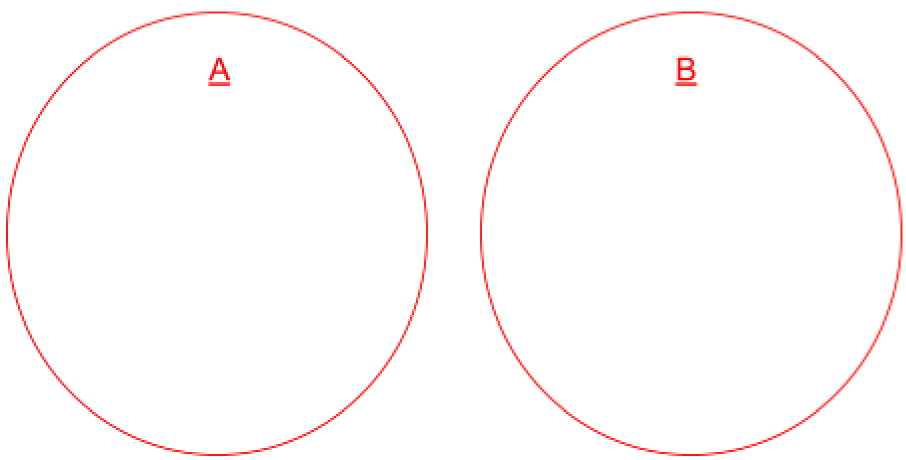
a. Two sets are *disjoint*, if they have **no elements in common**. In other words, two the sets A and B are disjoint if and only if

$$A \cap B = \varnothing$$

b. Give an example of two disjoint sets.

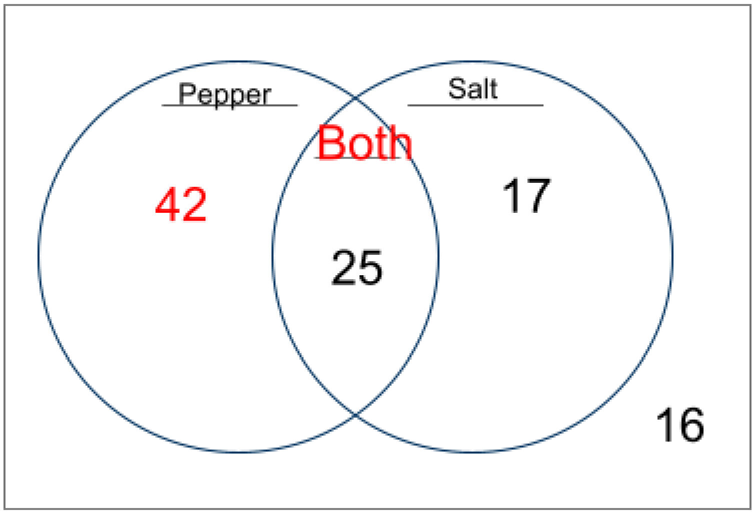
One example: $A=\{1, 3, 5\}$, $B=\{2, 4, 6\}$, $A \cap B = \varnothing$

c. What would the Venn diagram look like for two disjoint sets A and B? Draw the corresponding Venn diagram.



<u>Problem 5:</u> Interpreting Venn Diagram

a. Marcus asked 100 steak lovers whether they liked to put salt and pepper on their filet mignons.



i. *Fill in the missing pieces of the Venn Diagram above.*

ii. *Based on the Venn Diagram, how many put:*

1. *Salt:* 42
2. *Salt Only:* 17
3. *Pepper Only:* 42
4. *Salt and Pepper:* 25
5. *Pepper:* 67
6. *Neither:* 16

Next Time: We saw that we can use the special notations we've learned so far to identify the sections of a Venn Diagram. Next time, we'll dig deeper into the connections between sets and Venn Diagrams to learn about the Inclusion-Exclusion Principle.

Challenge Questions

1. *Let A be the set of all the even numbers, a.k.a. The integers divisible by 2. Let B be the set of all the integers divisible by 3. What is* $A \cap B$*?*

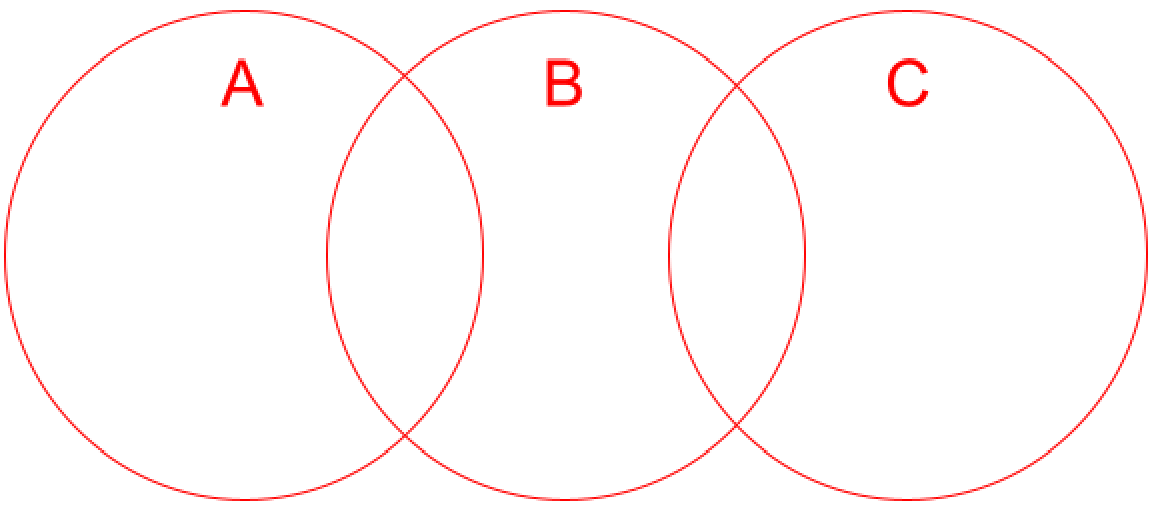
A ∩ B is the set of all integers divisible by 6

2. $S_1 = \{C, A, T\}$ and $S_2 = \{A, C, T\}$. *What is* $S_1 \cup S_2$?

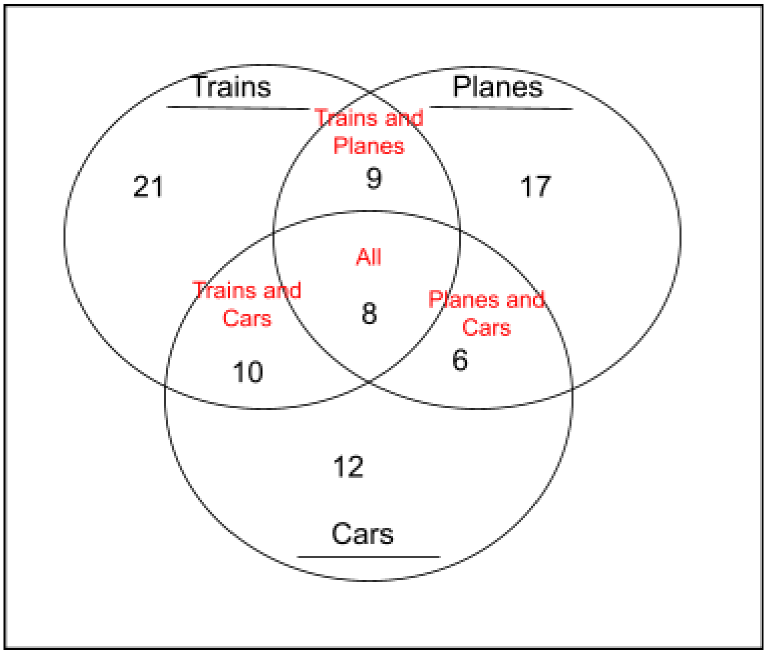
   $S_1 \cup S_2 = \{C, A, T\}$

3. *Draw the corresponding Venn diagram for:*

$$A \cap B \neq \varnothing$$
$$B \cap C \neq \varnothing$$
$$A \cap C = \varnothing$$



4. *Greg asked 100 kids whether they were collecting die-cast models of cars, trains, and airplanes.*



a. *Fill in the missing pieces of the Venn Diagram above.*

b. *Based on the Venn Diagram, how many put:*

   i. *Trains:* 48

   ii. *Planes:* 40

   iii. *Trains and Planes:* 17

   iv. *Trains and Planes, but not Cars:* 9

   v. *Trains and Cars, but not planes:* 10

   vi. *Neither of them :* 17

   vii. *All of them:* 8