Sets and Venn Diagrams Part 2

UCLA Olga Radko Math Circle Beginners 2

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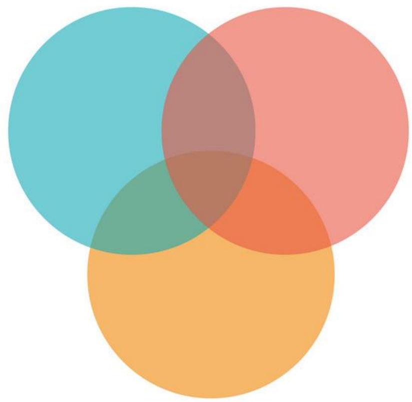

Warm-up:

Consider the following sets:

$$S_1 = \{ 1, 3, 5, \text{cat}, \text{dog}, \text{lion}, \text{pancake}\}$$
$$S_2 = \{ 4, 5, \text{waffles}, \text{dog}, 2, 1\}$$

a. Can you make a new set that contains elements that are found both in A and B?

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b. Can you make a new set that contains elements that are found in either A or B?

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<u>Problem 1:</u> Union and Intersection

a. The set of the elements that belong to the sets A **and** B is called the **intersection** of A and B and is denoted as **A ∩ B**.

   i. *Using the warm-up problem, how can we denote our answer for a?*

   ____

   ii. Let us rewrite this definition completely in the math language.

$$A \cap B = \{x : x \in A \text{ and } x \in B\} \qquad (1)$$

   In this mathematical sentence, the colon reads as *such that*. Translating back into English, *the* _______________ *of the sets A and B is defined as the set of the elements x such that* ____________ *A and* _____________ *B.*

b. The following is the definition of the **union** of two sets, written down in the math language.

$$A \cup B = \{x : x \in A \text{ or } x \in B\} \qquad (2)$$

   i. Translate definition (2) into English.

   ____

   ____

   ____

   ii. Going back to our warmup problem, how would you denote the answer for b?

   ____

c. What is $A \cap \varnothing$ for any set A?

____

d. What is $A \cup \varnothing$ for any set A?

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e. *Give an example of two sets and of their union different from the ones used so far.*

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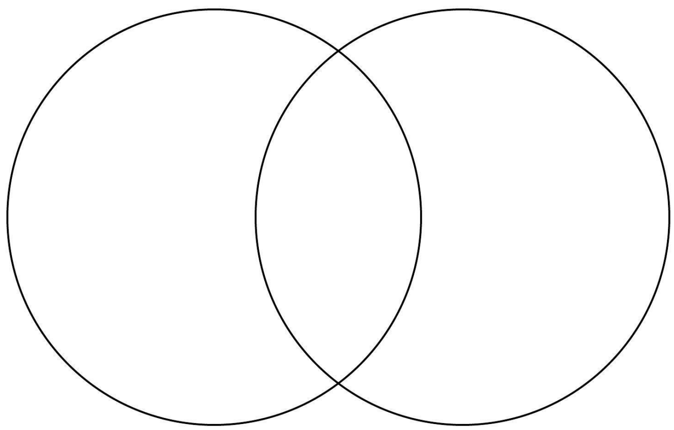
<u>Problem 2:</u> Venn Diagrams



a. Suppose we have two sets A and B. The *difference of the sets **A and B***, the set **A \ B**, is the *set of all the elements of the set A that* **do not belong** *to the set B*.

   i. *Label A \ B in the appropriate section of the Venn Diagram.*

   ii. *Translate B / A to English.*

   ____

   ____

   iii. *Label B \ A in the appropriate section of the Venn Diagram.*

   iv. *Using the notation we have learned so far, how would you label the middle of the Venn Diagram?*

   v. *Show the set A ∪ B on the Venn Diagram.*

b. *Let A be the set of spectators at a basketball game. Let B be the set of all the people at the game, spectators, coaches, staff, etc., wearing caps. Describe in your own words the set A \ B.*

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c. *[Challenge] Use the symbol ∉ to write the definition of the set A \ B in the math language. (Hint: Thinking about how we defined the definition for the union and intersection of two sets)*

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Red Hot Chilli Pepper Problem

a. *How many integers in the set S = {1, 2, 3, . . . , 98, 99, 100} are not divisible by 3?*

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b. *What is a set?*

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Problem 4: Disjoint Sets

a. Two sets are *disjoint*, if they have **no elements in common**. In other words, two the sets A and B are disjoint if and only if

$$A \cap B = \_\_\_\_\_\_\_\_\_\_$$

b. *Give an example of two disjoint sets.*

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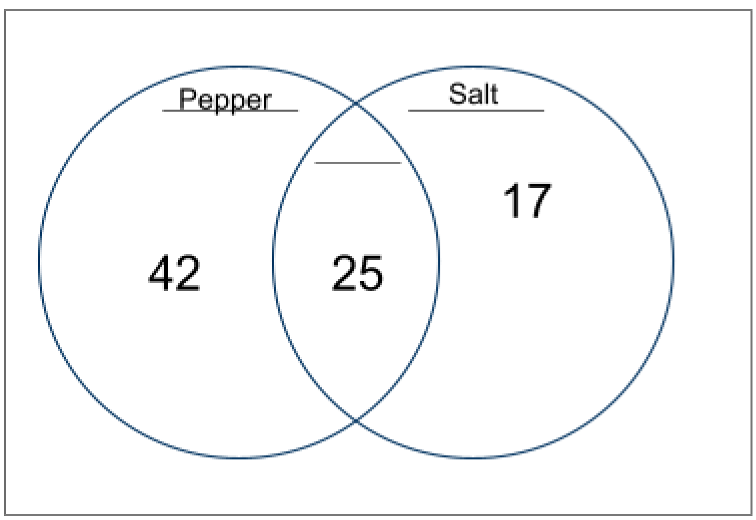
c. *What would the Venn diagram look like for two disjoint sets A and B? Draw the corresponding Venn diagram.*

<u>Problem 5:</u> Interpreting Venn Diagram

a. *Marcus asked 100 steak lovers whether they liked to put salt and pepper on their filet mignons.*



i. *Fill in the missing pieces of the Venn Diagram above.*

ii. *Based on the Venn Diagram, how many put:*

1. *Salt:* ____________
2. *Salt Only:* ____________

3. *Pepper Only:* ____________
4. *Salt and Pepper:* ____________
5. *Pepper:* ____________
6. *Neither:* ____________

Next Time: We saw that we can use the special notations we've learned so far to identify the sections of a Venn Diagram. Next time, we'll dig deeper into the connections between sets and Venn Diagrams to learn about the Inclusion-Exclusion Principle.

Challenge Questions

1. *Let A be the set of all the even numbers, a.k.a. The integers divisible by 2. Let B be the set of all the integers divisible by 3. What is $A \cap B$?*

   ____

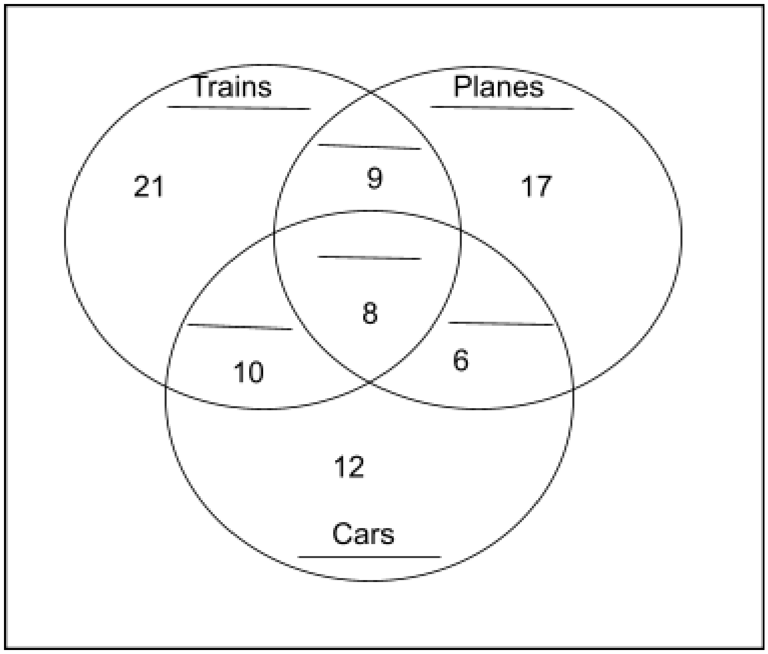
2. *$S_1 = \{C, A, T\}$ and $S_2 = \{A, C, T\}$. What is $S_1 \cup S_2$?*

   ____

3. *Draw the corresponding Venn diagram for:*

$$A \cap B \neq \varnothing$$
$$B \cap C \neq \varnothing$$
$$A \cap C = \varnothing$$

4. *Greg asked 100 kids whether they were collecting die-cast models of cars, trains, and airplanes.*



a. *Fill in the missing pieces of the Venn Diagram above.*

b. *Based on the Venn Diagram, how many put:*

   i. *Trains:* ____________

   ii. *Planes:* ____________

   iii. *Trains and Planes:* ____________

   iv. *Trains and Planes, but not Cars:* ____________

   v. *Trains and Cars, but not planes:* ____________

   vi. *Neither of them :* ____________

   vii. *All of them:* ____________