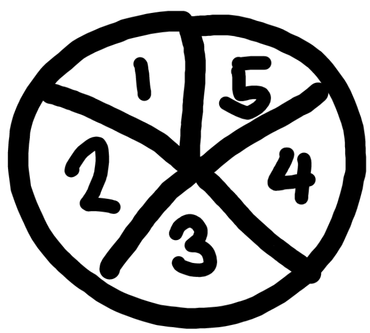
3. Making dartboards out of things that should not be dartboards

We can convert a probability problem into a dartboard problem and vice versa. Let S be a set of outcomes, and P a probability function that gives probabilities to each outcome in S.
On the flipped, we have a set of scores, and a dartboard with different regions on it. The link between these two is

The set of outcomes, S	$\Leftrightarrow$	The set of scores
P the probility function	$\Leftrightarrow$	The area of each region

Problem 18. What is the probability associated to each region on this dartboard?

$\frac{1}{5}$



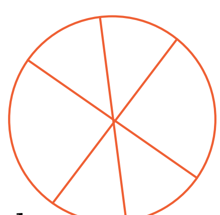
10 darts

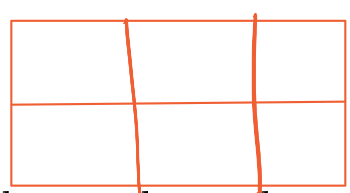
average

$\frac{1+2+3+4+5}{5} = 3$ pts

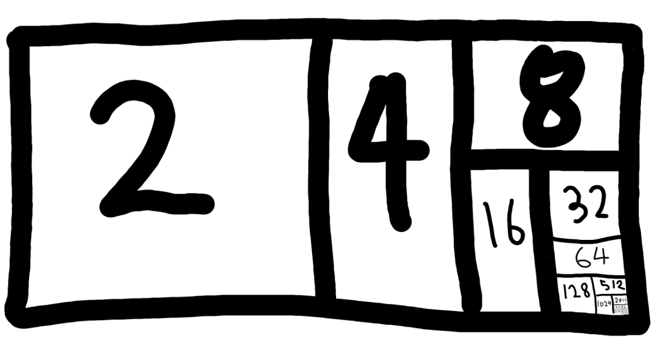
$3 \times 10 = \boxed{30 \text{pts}}$

Problem 19. Draw a dartboard whose scores give the probability of rolling die.





Problem 20. What is the average score when you throw a dart at these funny dartboards here? (Hint: convert it into a probability problem)

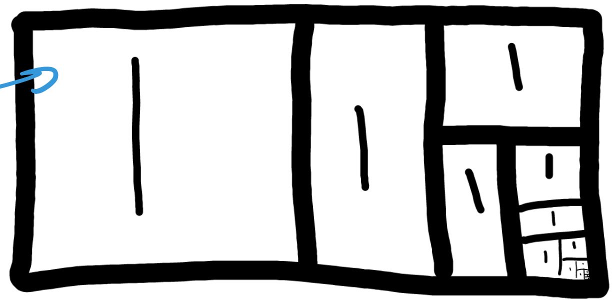


∞

$2 \cdot \frac{1}{2} + 4 \cdot \frac{1}{4} + 8 \cdot \frac{1}{8} \dots\dots 1+1+1+\dots$

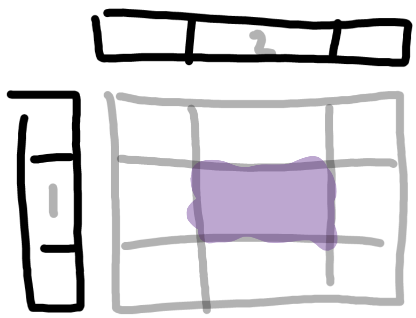
$1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{4} + 1 \cdot \frac{1}{8}$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} \ldots$



$\boxed{1}$

Problem 21. If we have two different dartboards, we can make a "mesh dartboard", but crossing them together. The probability of shooting it into region 1, and then into region 2 is given by the crossed regions. What is the probability of shooting it into the 1, and then the 2?



$\boxed{\frac{1}{9}}$

PROBABILITY II

BEGINNER CIRCLE 4/21/2013

1. Warm-up: Going Shopping

The math circle instructors go to the store to shop. The set of all items in the store will be called S, and there is a function that assigns a price to each item in the store, P. For instance

$$P(\text{Bazooka Bubblegum}) = .25$$

The price function can also assign prices to shopping carts that are full of items. For instance, Derek's shopping cart (which we will call S_{Derek}) contains Peanuts ($ 1.25), an episode of Seinfeld on DVD ($ 15), and Jonathan ($ 242.43). Therefore,

$$P(S_{\text{Derek}}) = 257.68$$

Problem 1. Morgan, Jeff and Isaac go shopping. ~~There~~ Their shopping carts contain

$$\begin{aligned} S_{\text{Morgan}} =& \{\text{Waterballoon, Waterbed, Watermelon, Watergun}\} = 10 \\ S_{\text{Jeff}} =& \{\text{Waterballoon, Watermelon, Pineapple}\} = 8 \\ S_{\text{Isaac}} =& \{\text{Waterbed, Watergun}\} = 7 \end{aligned}$$

Upon check out, we discover that

$$\begin{aligned} P(S_{\text{Morgan}}) =& 10 \\ P(S_{\text{Jeff}}) =& 8 \\ P(S_{\text{Isaac}}) =& 7 \end{aligned}$$

What is $P(\{\text{ Pineapple}\})$?

$P(S_{\text{Morgan}}) - P(S_{\text{Isaac}}) = 10 - 7 = 3$

wballoon, Wmelon

$5

Problem 2. Derek and Jonathan go to the store. Jonathan is planning on going to the beach, so he buys

$$S_{\text{Jonathan}} = \{ \text{ Swim Shorts, Surf Board, Snorkle}\} = 25$$

Jeff Hicks/Los Angeles Math Circle/UCLA Department of Mathematics .

Derek is a little more safety minded than Jonathan, so he buys everything that Jonathan does, plus a little extra.

$$S_{\text{Derek}} = \{\overset{25}{S_{\text{Jonathan}}}, \overbrace{\text{Helmet, Life Jacket, Flashlight}}^{10}\}$$

Suppose we additionally know on checkout that Helmets, Life Jackets and Flashlights cost a total of \$10, and $P(S_{\text{Jonathan}}) = 25$. What is $P(S_{\text{Derek}})$?

$\boxed{\$35}$

Problem 3. At Office Depot, Jeff and Rachel go shopping for school supplies. Because they have very different styles of mathematics, they buy very different supplies:

$$S_{\text{Jeff}} = \{\text{Whiteboard Marker, Pen, Stapler, Paper}\} = 10$$
$$S_{\text{Rachel}} = \{\text{Chalk, Pencil, Scotch Tape, } \underset{3}{\text{Paper}}\} = 7$$

Morgan can't decide which basket is better, so he buys all the items that are in Jeff's basket or in Rachel's basket. Suppose $P(S_{\text{Jeff}}) = 10$, and $P(S_{\text{Rachel}}) = 7$, and the price of paper is 3. What is $P(S_{\text{Morgan}})$?

$17 - 3 = \boxed{\$14}$

everything plus repeated paper

subtract

Problem 4. Jonathan and Isaac go shopping for dinner. Jonathan buys bananas (\$2), strawberries and cantaloupe and honeydew. Isaac buys strawberries, honeydew, boysenberry (\$ 4) and breadfruit (\$5). Derek is more picky than Jonathan and Isaac, so he buys the fruits that are in both baskets. We know that $P(S_{\text{Derek}}) = 4$ and $P(S_{\text{Jonathan}} = 13)$.

\$4

\$13 ← J = bananas (\$2), strawberry, cantelope, honeydew

I = straberry, honeydew, boysenberry (4), breadfruit (5)

D = $\underbrace{\text{strawberry, honeydew}}_{\$4}$

(i) What is $P(\text{Cantelope})$?

$\boxed{\$7}$

(ii) What is $P(S_{\text{Isaac}})$?

$\boxed{\$13}$

(iii) Suppose that Rachel now buys all the foods that are in Jonathan or Isaac's Basket. What is $P(S_{\text{Rachel}})$?

13

R= banana, straw, cant, honey, boys, bread

4 5

$26-4=\boxed{22}$

Problem 5. Suppose that Jeff and Derek go shopping, and Isaac buys everything that Jeff or Derek buy, and Morgan buys only things that both Jeff and Derek buy. Explain in a few sentences why

$$P(S_{\text{Isaac}}) = \underbrace{P(S_{\text{Derek}}) + P(S_{\text{Jeff}})}_{\text{everything plus repeats}} - P(S_{\text{Morgan}})$$

everything in D+J, w/out repeats

everything in both Derek + Jeff's bag (the repeats)

Problem 6. Suppose that Derek and Rachel and Morgan go shopping.

- Jeff only buys things in both Rachel's and Morgan's basket
- Isaac buys everything in either Jeff or Derek's Basket
- Jonathan buys everything in either Derek's or Rachel's basket
- Devon buys everything in either Derek's or Morgan's Basket
- Clyde buys everything in ~~either~~ both Jonathan's ~~or~~ and Devon's Basket.

Which two people necessarily bought the same items? Explain your solution in a couple of sentences or pictures.

Jeff = shared (Rachel + Morgan)

Isaac = Jeff + Derek = shared (Rachel + Morgan) + Derek

Jonathan = Derek + Rachel

Devon = Derek + Morgan

Clyde = shared (Jonathan + Devon)

shared (Derek + Rachel   Derek + Morgan)

Derek + shared (Rachel + Morgan)

Derek + Jeff

Clyde = Isaac

2. Sets!

A set is a group of things, none of which are repeated. In this section, we look at some relationships that sets can have with each other.

Definition 1. The **union** of two sets, $A \cup B$, is the set of all things that are in A or in B

Definition 2. The **intersection** of two sets, $A \cap B$, is the set of all things in A and in B

Definition 3. The size of a set A is the number of elements in it, and is written $|A|$.

Definition 4. The empty set $\emptyset$ is the set with no elements in it

Problem 7. Let $A = \{1, 3, 5, 4, 6\}$ and $B = \{1, 2, 3, 7, 8, \}$

(i) What is $A \cap B$

$\{1, 3\}$

(ii) What is $A \cup B$

$\{1, 2, 3, 4, 5, 6, 7, 8\}$

Problem 8. Let A be the set of all animals with fur, and B be the set of all animals that fly. Name some animals in $A \cap B$.

bats, flying squirrel?

Problem 9. Suppose that none of the elements in A are in B.

(i) What is $A \cap B$

$\emptyset$

(ii) What is $|A \cup B|$ in terms of $|A|$ and $|B|$

$|A \cup B| = |A| + |B|$

if there were overlaps... then

$|A \cup B| \leq |A| + |B|$

Problem 10. Let S denote the set of all rolls made with two dice. Let $S_{[2]}$ be the subset of all rolls that are a 2, and $S_{[4]}$ be the set of all rolls that have faces totaling to 4. For this question, when you want to say the roll where the first die shows a 2, and the second die shows a 5, write (2,5)

(i) What is the set of all dice that have faces totaling to 2 or 4?

$S_2 \cup S_4 = \{(1,1), (2,2), (3,1), (1,3)\}$

(ii) What is the size of that set?

4

Problem 11. Again, let S denote the set of all rolls made with two dice. Let S_{E1} be the set of all rolls where the first die has an even value, and S_{E2} be the set of all rolls where the second die roll has an even value.

(i) What is the set of all dice rolls where the first die **and** the second die have an even value.

$S_{E1} \cap S_{E2}$

(2,5), (2,4)... (1,6), (2,4)...

(ii) What is the size of that set?

2, 4, 6

$\underset{D_1}{3} \times \underset{D_2}{3} = \boxed{9}$

(iii) What is the set of all dice rolls where the first die **or** the second die have an even value?

$S_{E1} \cup S_{E2}$

(iv) What is the size of that set?

2 4 6

EO + OE + EE

3×3 3×3 3×3

9 9 9

$\boxed{27}$