

Graphs and Geometry

Los Angeles Math Circle

9 May 2021

1 Graphs

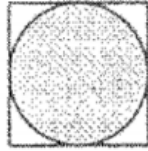
1. There are 100 cities in a country, and each road connects two of those cities. If each city has 4 roads going out of it, how many roads are there in total?
2. There are 15 cities in a country, each connected to at least 7 other cities by roads. Show that it is possible to drive from any city to any other city using the roads.
3. Prove that in any group of 6 people there are either 3 people who all know each other, or 3 people none of whom know each other. All acquaintances are assumed to be mutual. (Meaning that if John knows Pete, than Pete necessarily knows John)

4. $\triangle ABC$ is isosceles with $AB = BC$. It is known that one of the sides AB and AC is 7, and the other is 3. Which is which?

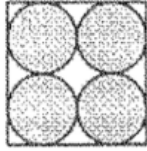
5. Let A, B, C be points on a circle ω . Let P be a point such that the line PB is tangent to ω . Also let A_1 be the foot of the altitude from P to AB , and C_1 be the foot of the altitude from P to CB . Show that $A_1C_1 \perp AC$.

3 Math Kangaroo Area Approximation

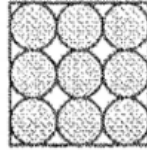
1. Each of the pictures below shows a square with a side of 1 and shaded circles. In which picture is the shaded area the greatest? Justify your answer.



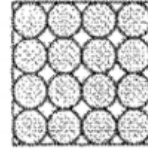
Picture 1



Picture 2

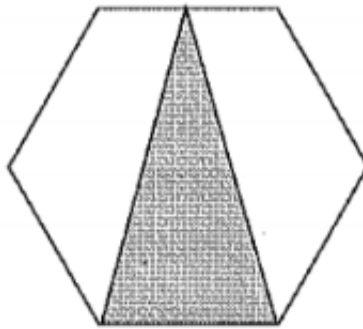


Picture 3

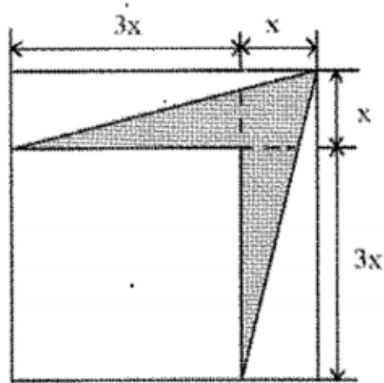


Picture 4

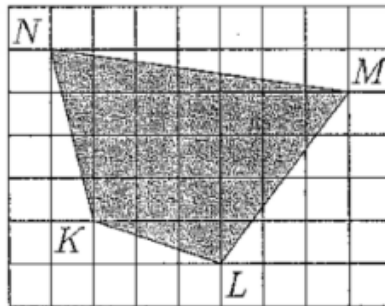
2. What is the ratio of the area of the shaded triangle to the area of the regular hexagon?



3. Find the area of the shaded portion with respect to x .



4. The diagram shows a shaded quadrilateral $KLMN$ drawn on a grid. Each cell of the grid has sides of length 2cm. What is the area of $KLMN$?



5. The large triangle shown in the picture was divided into 36 small equilateral triangles, each with an area of 1. Find the area of triangle ABC .

