## 2. THINGS THAT ARE NOT DARTS

How do we describe probability? Probability somehow measures the likelihood that a specific event occurs out of a whole bunch of different events. For example, if we flip a coin, we have 2 different possible outcomes:

$$S = \{\text{heads}, \text{tails}\}$$

The probability that we flip a head is the likelihood that we pick heads out of that set.

**Definition 1.** The set of outcomes of some probability problem is written with the letter $S$. For every possible outcome $x$, we can assign a number telling us how likely that event is to occur, called the probability of $x$, and written $P(x)$

The function $P(x)$ takes an outcome $x$ and assigns a probability to them. For instance,

$$P(\text{heads}) = \frac{1}{2}$$

means the likelihood of flipping a head is 1 in 2.
The probability function follows two special rules:

(i) Let $a$ and $b$ be two separate outcomes in $S$. Then the probability of picking outcome $a$ is $P(a)$, while the probability of picking outcome $b$ is $P(b)$. The probability of picking either outcome $a$ or $b$ is

$$P(a \text{ or } b) = P(a) + P(b)$$

(ii) Let $S = \{s_1, s_2, \ldots, s_n\}$ be the possible events of a probability problem. Then the total probability of picking an outcome from $S$ is

$$P(s_1 \text{ or } s_2 \text{ or } \ldots \text{ or } s_n) = 1$$

This is like saying, when you flip a coin, you have a probability of $\frac{1}{1}$ of getting either heads or tails.

**Problem 8.**

(i) What is the set of outcomes $S$ for flipping a coin?

$S = \{\text{heads}, \text{tails}\}$

(ii) If the coin is fair, then the chances of getting heads or tails is the same. Can you write this down using $P(\text{heads})$ and $P(\text{tails})$?

$P(\text{heads}) = P(\text{tails})$

(iii) Since the only possible outcomes are heads and tails, what does $P(\text{heads}) + P(\text{tails})$ equal?

1

(iv) Using the two earlier sections, use some algebra to show $P(\text{heads}) = \frac{1}{2}$

P(heads) = P(tails)

P(heads) + P(tails) = 1

P(heads) + P(heads) = 1

$\therefore$ P(heads) = $\frac{1}{2}$

**Problem 9.**

(i) When a die is rolled, what is the set $S$ of possible outcomes showing on the die?

$S = \{1, 2, 3, 4, 5, 6\}$

(ii) If the die is fair, then the chances of rolling different numbers is the same. Can you write this down using $P(1), P(2), \ldots P(6)$, the probabilities that you roll a $1, 2, 3, \ldots 6$?

P(1) = P(2) = P(3) = P(4) = P(5) = P(6)

(iii) Since the only possible outcomes are $1, 2, 3 \ldots$ or $6$, what does that tell us

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = ?$$ 1

(iv) Using the two earlier sections, conclude that $P(1) = \frac{1}{6}$

all equal eachother: $P(1) = P(2) = P(3) = P(4) = P(5) = P(6)$

and added together $= 1$

so $P(1) \cdot 6 = 1$ $\therefore$ $\boxed{P(1) = \frac{1}{6}}$

Notice that the above problems rely on the fact that you know that the outcomes are equally likely. However, sometimes the outcomes are not equally likely!

**Problem 10.** If you flip two coins, and you can tell them apart, then there are four different possible outcomes,

$$S = \{HH, HT, TH, TT\}$$

(i) What is the probability that you get two heads? Explain your solution using full sentences.

Probability of getting H the 1st time $= \frac{1}{2}$

Probability of getting H the 2nd time $= \frac{1}{2}$

so $\frac{1}{2} \times \frac{1}{2} = \boxed{\frac{1}{4}}$ *you can also see HH is $\frac{1}{4}$ outcomes in the set

(ii) What is the probability that you get a head and a tail (this can happen two different ways!). Explain your solution in full sentences

a $\frac{2}{4} = \frac{1}{2}$ because 2 ways to get this in the set

+ $P(HT) \times P(TT) + P(TT) \times P(HT)$

$\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \boxed{\frac{1}{2}}$

(iii) What is the probability that you do not get 2 heads? Explain your solution in full sentences.

3 ways for this to happen out of 4 in the set

$\boxed{\frac{3}{4}}$

**Problem 11.**

(i) Two dice are rolled. How many different outcomes are there? (note: if you roll a 5 and a 6, it is different than rolling a 6 and 5!)

Combinatorics! $6 \times 6 = \boxed{36}$

(ii) How many ways can you can you roll a 2? *from 2 die

1 way 1 + 1

(iii) How many ways can you roll a 3?

2 ways 1 + 2
2 + 1

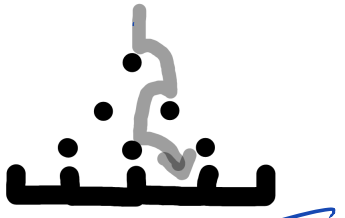
(iv) Explain, in full sentences, why $2 \times P(2) = P(3)$? (The probability of rolling a 3 is twice as much as rolling a 2).

$P(2) = \frac{1}{36}$ $P(3) = \frac{2}{36} = \frac{1}{18}$

(v) What is the probability of rolling a 3?

$\frac{1}{18}$

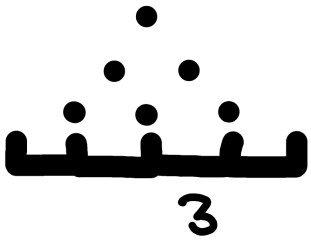
**Problem 12.** A pachinko machine is set up, and balls bounce from the top of the machine to the bottom. Suppose the probability of the ball taking any path from the top to the bottom is the same. What is the probability of the ball falling along the given path? (Hint: How many paths are there from the top to the bottom)



8 paths

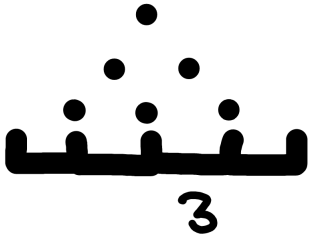
$\frac{1}{8}$

**Problem 13.** In the pachinko machine above, what is the probability that the ball falls into the bin labeled 3? How did you arrive at your solution?




$\frac{3}{8}$

**Problem 14.** What is the probability that the ball does not fall into the bin labeled 3? How did you arrive at your solution?




$\frac{5}{8}$

I subtracted ☺

**Problem 15.** Suppose that we roll a 6 sided die. Then the number of outcomes that die can roll is 6. What is the number of outcomes of rolling a blue die and a red die? What is the number of outcomes rolling a red and a blue and a green die?

blue & red = $6^2 = 36$

R, B, G = $6^3 = 216$

**Problem 16.** If you roll 2 die, what is the probability that you roll a 8?

4 + 4
5 + 3
3 + 5
2 + 6
6 + 2

$\frac{5}{36}$

**Problem 17.** If you roll 3 die, what is the probability that you roll a 8?

1 + 1 + 6 → 3 ways
1 + 2 + 5 → 6 ways
1 + 3 + 4 → 6 ways
2 + 2 + 4 → 3 ways
2 + 3 + 3 → 3 ways

$\frac{21}{216} \rightarrow \frac{7}{72}$