

# Lesson 4: Stars and Bars

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**Problem 1** (Binomial formula).

Show that

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n-1}ab^{n-1} + b^n$$

**Problem 2.**

Given an exam with three problems, how many ways are there to assign positive point values to each problem so that the whole exam adds up to 100 points?

**Problem 3.**

Find the number of ways to write a positive integer  $n$  as an ordered sum of  $k$  positive integers. Here “ordered” means that  $3 = 1 + 2$  and  $3 = 2 + 1$  would be different representations of 3 as a sum of 2 numbers.

a) Simply show us the correct formula, no proof needed.

b) Now prove your formula.

Hint: First, pick some small values of  $n$  and  $k$ , and compute the answer in those specific examples. Use these examples to formulate a conjecture about how the answer depends on  $n$  and  $k$  (it might be useful to look for the answers you get in small cases in the Pascal’s triangle). Once you have a formula which seems to work in specific cases, try to find a general proof for it.

**Problem 4.**

a) Find a number  $u > 1$  which occurs in the Pascal’s triangle at least 4 times.

b) Find a number  $u > 1$  which occurs in the Pascal’s triangle at least 5 times.

**Problem 5.**

Let  $AA_1$  and  $BB_1$  be altitudes in a triangle  $\triangle ABC$ . Show that  $CA_1 \cdot CB = CB_1 \cdot CA$ .

**Problem 6.**

Let  $K$  and  $N$  be points on the sides  $AB$  and  $AC$  of the triangle  $\triangle ABC$  such that  $AK = KB$  and  $AN = 2NC$ . Let  $P$  be the intersection of  $NK$  with the median  $AM$ . Find the ratio  $AP/PM$ .