Lesson 4: Stars and Bars

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Problem 1.
How many ways are there to arrange 2 green, 2 red and 2 blue balls in a row so that no two balls of the same color are adjacent to each other?

Problem 2.
Compute

a)

$$1 + \binom{10}{1}3 + \binom{10}{2}3^2 + \binom{10}{3}3^3 + \cdots + \binom{10}{8}3^8 + \binom{10}{9}3^9 + 3^{10}.$$

b)

$$2^{10} - \binom{10}{1}3 \times 2^9 + \binom{10}{2}3^2 \times 2^8 - \cdots + \binom{10}{8}3^8 \times 2^2 - \binom{10}{9}3^9 \times 2 + 3^{10}.$$

Problem 3.
a) Given an exam with two problems, how many ways are there to assign positive point values to each problem so that the whole exam adds up to 100 points?

b) Given an exam with **three** problems, how many ways are there to assign positive point values to each problem so that the whole exam adds up to 100 points?

Problem 4.
Find the number of ways to write a positive integer n as an ordered sum of k positive integers. Here "ordered" means that $3 = 1 + 2$ and $3 = 2 + 1$ would be different representations of 3 as a sum of 2 numbers.
a) Simply show us the correct formula, no proof needed.

b) Now prove your formula.

Problem 5.
a) Find a number $u > 1$ which occurs in the Pascal's triangle at least 4 times.

b) Find a number $u > 1$ which occurs in the Pascal's triangle at least 5 times.

Problem 6.
Let AA_1 and BB_1 be altitudes in a triangle $\triangle ABC$. Show that $CA_1 \cdot CB = CB_1 \cdot CA$.

Problem 7.
Let K and N be points on the sides AB and AC of the triangle $\triangle ABC$ such that $AK = KB$ and $AN = 2NC$. Let P be the intersection of NK with the median AM. Find the ratio AP/PM.