Warm-up:

Consider the following sets:

\[ S_1 = \{1, 3, 5, \text{cat, dog, lion, pancake}\} \]
\[ S_2 = \{4, 5, \text{waffles, dog, 2, 1}\} \]

a. Can you make a new set that contains elements that are found both in \( A \) and \( B \)?

_______________________________________________________________________

b. Can you make a new set that contains elements that are found in either \( A \) or \( B \)?

_______________________________________________________________________
**Problem 1: Union and Intersection**

a. The set of the elements that belong to the sets A and B is called the **intersection** of A and B and is denoted as $A \cap B$.
   
i. *Using the warm-up problem, how can we denote our answer for a?*

   _____________________________________________________________________

   ii. Let us rewrite this definition completely in the math language.

   $$A \cap B = \{x : x \in A \text{ and } x \in B\} \quad (1)$$

   In this mathematical sentence, the colon reads as *such that*. Translating back into English, *the ______________ of the sets A and B is defined as the set of the elements x such that ___________ A and ____________ B.*

b. The following is the definition of the **union** of two sets, written down in the math language.

   $$A \cup B = \{x : x \in A \text{ or } x \in B\} \quad (2)$$

   i. *Translate definition (2) into English.*

   _____________________________________________________________________
   _____________________________________________________________________
   _____________________________________________________________________

   ii. *Going back to our warmup problem, how would you denote the answer for b?*

   _____________________________________________________________________

   c. *What is $A \cap \emptyset$ for any set A?*

   _____________________________________________________________________

   d. *What is $A \cup \emptyset$ for any set A?*

   _____________________________________________________________________
e. Give an example of two sets and of their union different from the ones used so far.

Problem 2: Venn Diagrams

a. Suppose we have two sets A and B. The difference of the sets $A$ and $B$, the set $A \setminus B$, is the set of all the elements of the set $A$ that do not belong to the set $B$.

i. Label $A \setminus B$ in the appropriate section of the Venn Diagram.

ii. Translate $B \setminus A$ to English.

iii. Label $B \setminus A$ in the appropriate section of the Venn Diagram.

iv. Using the notation we have learned so far, how would you label the middle of the Venn Diagram?

v. Show the set $A \cup B$ on the Venn Diagram.
b. Let A be the set of spectators at a basketball game. Let B be the set of all the people at the
game, spectators, coaches, staff, etc., wearing caps. Describe in your own words the set A \ B.

_______________________________________________________________________

c. [Challenge] Use the symbol $\notin$ to write the definition of the set A \ B in the math language.
(Hint: Thinking about how we defined the definition for the union and intersection of two
sets)

_______________________________________________________________________

Red Hot Chilli Pepper Problem

a. How many integers in the set $S = \{1, 2, 3, \ldots, 98, 99, 100\}$ are not divisible by 3?

_______________________________________________________________________

b. What is a set?

_______________________________________________________________________

Problem 4: Disjoint Sets

a. Two sets are disjoint, if they have no elements in common. In other words, two the sets
A and B are disjoint if and only if

$$A \cap B = \text{________________________}$$

b. Give an example of two disjoint sets.

_______________________________________________________________________
c. What would the Venn diagram look like for two disjoint sets A and B? Draw the corresponding Venn diagram.

Problem 5: Interpreting Venn Diagram

a. Marcus asked 100 steak lovers whether they liked to put salt and pepper on their filet mignons.

i. Fill in the missing pieces of the Venn Diagram above.

ii. Based on the Venn Diagram, how many put:

1. Salt: ____________
2. Salt Only: ____________
3. Pepper Only: ____________
4. Salt and Pepper: ____________
5. Pepper: ____________
6. Neither: ____________

Next Time: We saw that we can use the special notations we’ve learned so far to identify the sections of a Venn Diagram. Next time, we’ll dig deeper into the connections between sets and Venn Diagrams to learn about the Inclusion-Exclusion Principle.

Challenge Questions

1. Let $A$ be the set of all the even numbers, a.k.a. The integers divisible by 2. Let $B$ be the set of all the integers divisible by 3. What is $A \cap B$?

_______________________________________________________________________

2. $S_1 = \{C, A, T\}$ and $S_2 = \{A, C, T\}$. What is $S_1 \cup S_2$?

_______________________________________________________________________

3. Draw the corresponding Venn diagram for:

\[
\begin{align*}
A \cap B & \neq \emptyset \\
B \cap C & \neq \emptyset \\
A \cap C & = \emptyset
\end{align*}
\]
4. Greg asked 100 kids whether they were collecting die-cast models of cars, trains, and airplanes.

![Venn Diagram with numbers]

a. Fill in the missing pieces of the Venn Diagram above.

b. Based on the Venn Diagram, how many put:

i. Trains: ____________

ii. Planes: ____________

iii. Trains and Planes: ____________

iv. Trains and Planes, but not Cars: ____________

v. Trains and Cars, but not planes: ____________

vi. Neither of them: ____________

vii. All of them: ____________