WORKING TOGETHER

MATH CIRCLE (BEGINNERS) 11/20/2011

1) Peter Piper picked six pecks of pickled peppers in six minutes. His friends Peri, Puck, Patty, Penny, and Pip all pick pecks of pickled peppers at the same rate. If the six of them work together, how long will it take them to pick six pecks of pickled peppers?

Solution: Peter Piper picks six pecks in six minutes, so he picks one peck in one minute. So do each of his 5 friends. So to pick six pecks, the six of them working together need just **one minute**.

2) 80 men can harvest 160 fields in 40 days. How long will it take 40 men to harvest 20 fields?

Solution: 80 men harvest 160 fields in 40 days; this means each man can harvest two fields in 40 days; so 1 man can harvest 1 field in 20 days. Therefore 40 men harvest 40 fields in 20 days, so they can do half that–20 fields–in half the time–namely, **10 days**.

3) Ian, Cor, and Dan Quayle are peeling potatoes. Ian peels 4 potatoes per minute, Cor peels 6 potatoes per minute, and Dan Quayle peels 10 potatoes per minute. If they work together, how long will it take them to peel 2875 potatoes?

Solution: Working together, they peel a total of 4+6+10 = 20 potatoes per minute. In order to peel 2875 potatoes, they will need 2875 (potatoes) / 20 (potatoes per minute) = **143.75 minutes** = 143 min 45 sec.

4) If Graham can paint a room in 4.5 hours, how much of a room can he paint in 1 hour? (This is not a trick question!)

Solution: In one hour, he can paint a 1/4.5 fraction of a room, which is **2/9 of the room** (or about 22%).

5) Maya and Kathy have 22 jigsaw puzzles to do. Maya can finish one puzzle in 12 minutes, and Kathy can do it in 10 minutes. Working together, how long will it take them to finish all?

Solution: In one hour, Maya finishes 5 puzzles, and Kathy finishes 6 puzzles, so together they finish 11. Thus to finish 22 puzzles, they need **2 hours**.

6) Kaitlin and Eric are housepainters. Kaitlin paints one house every 3 days, and Eric paints one every 4 days. Working together, how long will it take them to paint 20 houses? (They're allowed to be painting the same house at the same time!)

Solution: In one day, Kaitlin paints 1/3 of a house and Eric paints 1/4 of a house. So in total, they paint

$$\frac{1}{3}+\frac{1}{4}=\frac{4}{12}+\frac{3}{12}=\frac{7}{12}$$

of a house. In order to paint 20 houses, they will need 20 (houses) / 7/12 (houses per day) = 240/7 days = **34 2/7 days**.

7) Albert, Holden, and Maria work at a landscaping company that's been hired to mow all the school district's football fields. Albert, using his old-fashioned push-mower, can mow a football field in 2 days. Holden, using a newer self-propelled lawnmower, can do it in just 1.5 days. Maria, on her state-of-the-art riding mower, can mow up to 3 football fields in a single day! Working together, how long will it take them to mow all 10 of the district's football fields? (They're allowed to be mowing the same field at the same time!)

Solution: In a single day, Albert mows 1/2 of a field, Holden mows 1/1.5 = 2/3 of a field, and Maria mows 3 fields. Together, they mow 1/2 + 2/3 + 3 = 3/6 + 4/6 + 18/6 = 25/6 of a field each day. Thus to mow 10 fields, they need 10 (fields) / 25/6 (fields per day) = 60/25 days = 12/5 days = **2.4 days**.

8) Oh no! The rowboat's just hit a **HUGE ROCK!** The water's rushing in at a rate of 1 gallon per second! The boaters will have to empty the water with buckets while they try to make it back to land. Daniel can scoop a bucket of water and toss it back in the lake every 3 seconds, and Michael, an expert bucket-thrower, can do it every 2 seconds. The boat can hold at most 25 gallons before it starts to sink. If each bucketful is one gallon, and William, the fastest rower, can make the boat go 3 feet per second, what is the farthest distance the shore could be so that they still make it back before going underwater?

Solution: Every 6 seconds, the boat gains 6 gallons of water flooding in. At the same time, Daniel scoops out 2 gallons of water, and Michael scoops out 3 gallons of water. So the water amount is increasing at a rate of of 1 gallon per 6 seconds, or 1/6 gal/sec. The boat can hold 25 gallons, so they have 25 (gal) / 1/6 (gal/sec) = 25*6 sec = 150 sec. to reach shore. Since William rows at 3 feet per second, he can row up to 3 (feet per second) * 150 (seconds) = **450 feet** to reach shore.

9) If you know some algebra, try these: (a) Zhubeen and Sam are fence-painters. If Zhubeen takes a hours to paint a fence, and Sam takes b hours, then how many hours does it take them, working together to paint one fence.

Solution: In one hour, they paint 1/a and 1/b of a fence, respectively. Hence together they paint 1/a + 1/b = (a+b)/ab. To paint 1 fence, they require 1 (fence) / (a+b)/ab (fences / hr) = ab/(a+b) hours.

(b) Michelle, Scott, and George are street-line painters. If Michelle takes x hours to paint the lines on one mile of street, Scott takes y hours to paint one mile of street, and George takes z hours to paint one mile, how many miles can they paint, working together, in 3 hours?

Solution: In one hour, they paint 1/x, 1/y, and 1/z of a mile, respectively, so together they paint 1/x + 1/y + 1/z fraction of a mile, which is (yz + xz + xy)/xyz of a mile. If they work for 3 hours, they paint 3(yz + xz + xy)/xyz miles worth of road.