

# PROBABILITY I

BEGINNER CIRCLE 4/14/2013

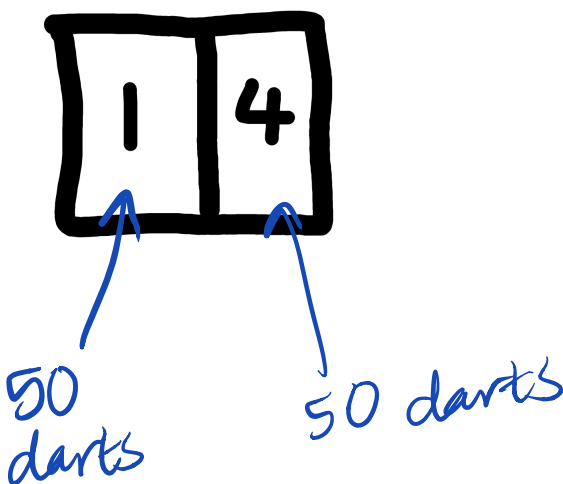
## 1. WARM-UP: PLAYING DARTS

I hope that you are all familiar with the game of darts. The game goes something like this: A board is set up on the opposite side of the room, with different regions corresponding to different amounts of points. Darts are thrown across the room, and the number of points that you earn is equal to the number on the region the dart lands in.

The math instructors want to play darts. As they are all mathematicians, they have horrible vision, and thus do not play darts very well. The best that they can do is throw darts in such a way that they know that will hit the dart board, but they have no idea *where* on the dartboard the dart will strike. Furthermore, they have a pretty hard time making out where the dart landed when it strikes the other side of the wall.

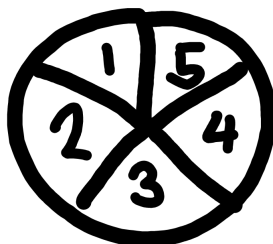
Fortunately, our protagonists are not very picky, and do not really care what the exact score of the game was, but rather, are ok making guesses about the score that they get. For each of the following games, give a rough estimate of the score at the end of shooting darts.

**Problem 1.** Suppose Isaac throws a hundred darts at this dartboard. What is a good guess for his score?



$$50(1) + 50(4) = 50 + 200 = 250$$

**Problem 2.** Suppose Derek throws 10 darts at this dartboard. What is a good guess for his score.



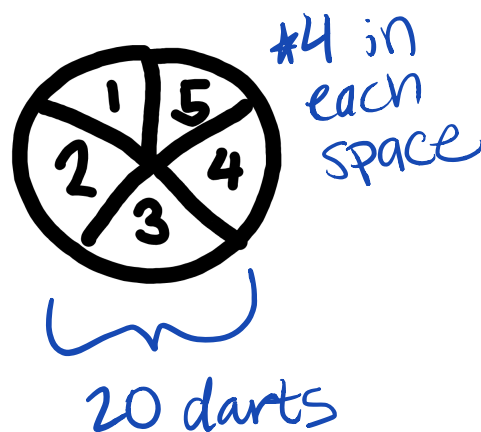
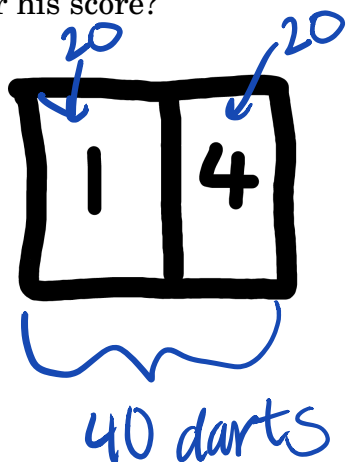
$$\frac{10}{5} \rightarrow \begin{array}{l} \text{\#throws} \\ \hline \text{\# places to land} \end{array}$$

$$2 \cdot [1 + 2 + 3 + 4 + 5]$$

$$2 \cdot [15]$$

$$\boxed{30}$$

**Problem 3.** Morgan is a lefty, so  $\frac{2}{3}$  of his darts go to the dartboard on the left, while the remaining  $\frac{1}{3}$  go to the dartboard on the right. If he throws 60 darts, what is a good guess for his score?



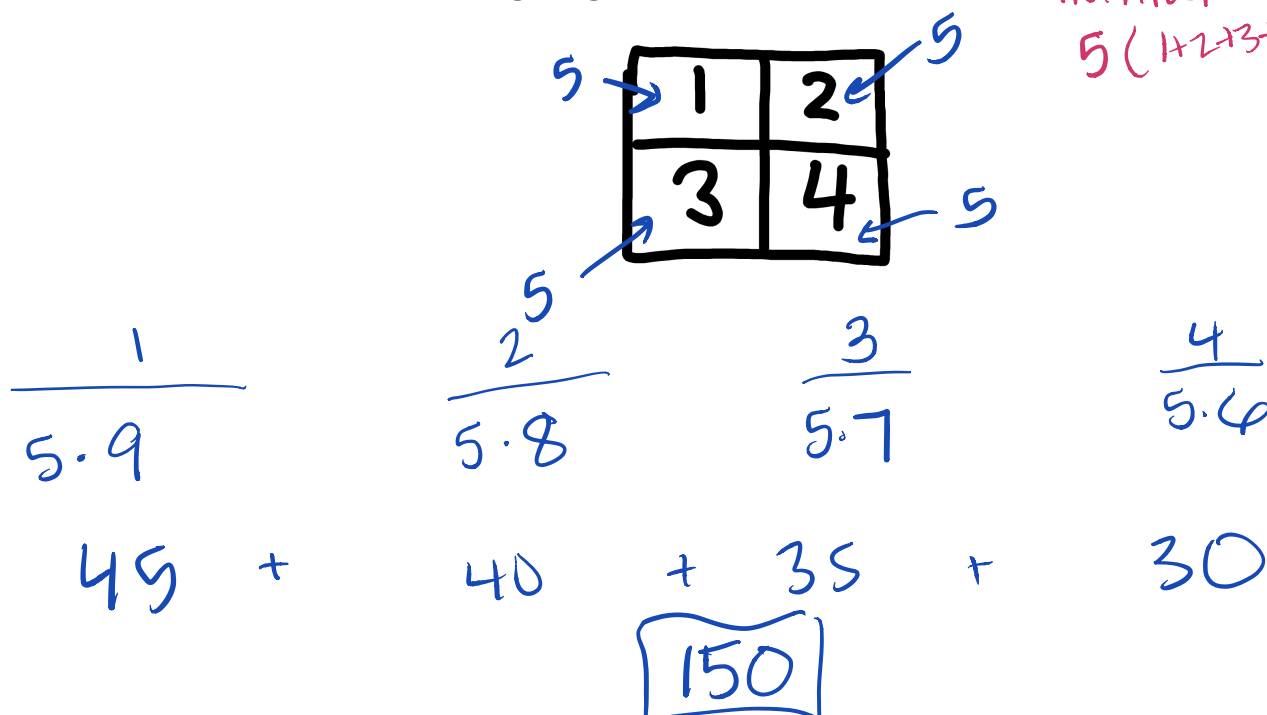
$$20 \cdot 1 + 20 \cdot 4 + 4 \times (1 + 2 + 3 + 4 + 5)$$

$$20 + 80 + 4 \times 15 = \boxed{140}$$

**Problem 4.** Jeff plays “misère” darts, which is to say that he cheats. Whenever Jeff throws a dart, he gives himself the score of all the other regions that he missed. If he

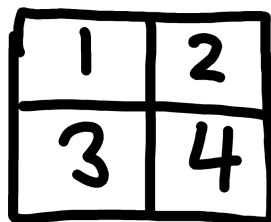
throws 20 darts, what is a good guess for his final score?

truthful  
 $5(1+2+3+4) = 50$



**Problem 5.** After years of training in a monastery in Japan, Jonathan has perfected the art of “nihon bo shuriken”, which allows him to throw two darts at the same time. When he throws the two darts, the score that he gets is the *product* of the scores in the two regions he hits. If he throws  $2 \times 40 = 80$  darts, what is a good guess for his score? *permutation*

List			
$D_1$	$D_2$		
1	1=1	2	1=2
2	2=4	2	3=6
3	3=9	2	4=8
4	4=16	3	1=3
1	2=2	3	2=6
1	3=3	3	4=12
1	4=4	4	1=4
		4	2=8
		4	3=12



$\frac{4}{D_1} \times \frac{4}{D_2} = 16$

40 turns  
 16 possible ways

$\frac{\# \text{throws}}{\# \text{places to land}} \rightarrow \frac{40}{16}$

$\frac{40}{16} \times \text{sum of all possible scores}$

$\frac{40}{16} \times 100 = 10 \cdot 25 = 250$

**Problem 6.** Isaac is playing hardcore darts. In hardcore darts, you throw two darts per turn. You get the points only if you throw both darts into the same region. Suppose Isaac throws  $2 \times 90$  darts at this dartboard. What is a good guess for his score at

example: must throw 2 darts into 6 to score 6.  
the end of the game.

1	2	3
4	5	6
7	8	9

scoring  
Darts =  $90 \times \frac{1}{9} = 10$

$$\underbrace{1+2+3+4+5+6+7+8+9}_{45} + \frac{45}{9} + 5 = 50$$

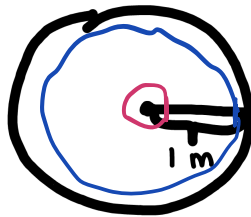
$$\frac{9}{D_1} \times \frac{9}{D_2} = 81 \text{ total}$$

to score:  $\frac{9}{D_1} \times \frac{1}{D_2} = 9$  spots where you score

$$\frac{9}{81} = \frac{1}{9}$$

**Problem 7.** Jeff and Derek begin designing a new dartboard. Because their markers ran out of ink, the best they can do is cut out a large circle, 1 meter in radius. They decide that the number of points that each dart is worth is equal to the distance the dart is away from the edge—(1 – the distance from the center). If they throw 20 darts at this dartboard, what is a good guess for their score at the end of the game?

with the info given, we can't do it!!



$$A_{\text{circle}} = \pi r^2$$

$$\text{Circumference} = 2\pi r$$

we need calculus

can we estimate?

estimate using areas?