

Lesson 2.5: Combinations and Pascal's Triangle

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1 From Last Week

Problem 2.

Show that if $n \geq 4$ and $2 \leq k \leq n - 2$, then

$$\binom{n}{k} = \binom{n-2}{k-2} + 2\binom{n-2}{k-1} + \binom{n-2}{k}$$

Problem 4.

a) Show that if p is prime and $1 \leq k < p$, then

$$p \mid \binom{p}{k}$$

b) Show that if p is prime, then

$$p \mid \binom{2p}{p} - 2$$

Problem 5.

Let $ABCD$ be a cyclic quadrilateral, and let T be the intersection of lines AB and CD . Assume A lies on the segment TB and D lies on the segment TC . Show that $TA \cdot TB = TC \cdot TD$.

Problem 6.

Let BB_1 and CC_1 be altitudes in a triangle $\triangle ABC$. Show that the tangent line at A to the circumcircle of $\triangle ABC$ is parallel to B_1C_1 .

2 New Problems

Problem 1.

Toys R Us has recently introduced a new revolutionary type of toy – a wire cube with a colored sphere at each corner. The spheres can be one of 8 colors, and each cube has to contain all 8 possible colors. How many different cubes can Toys R Us produce?

Problem 2.

Count the number of 5-digit numbers which contains exactly the digits 1,2,3,4,5 and the even digits are not adjacent to each other.

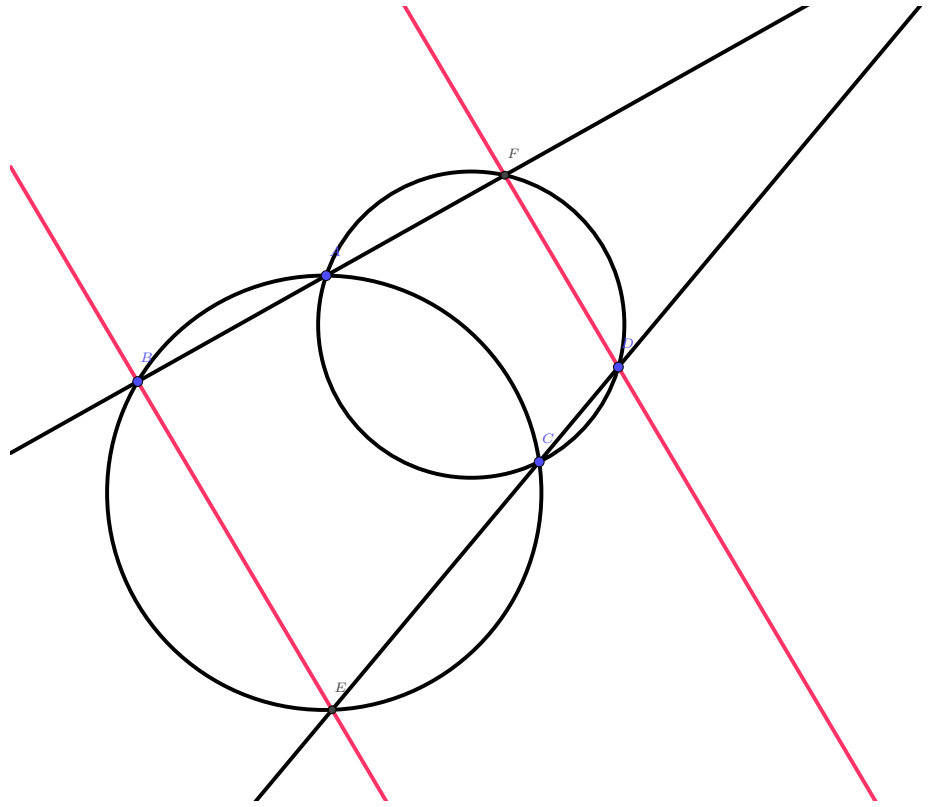


Figure 1:

Problem 3.

Let circle P and circle Q be intersecting at points A and C. Take the line AB such that B is on circle P and AB intersects F on circle Q. Likewise, take the line EC such that E is on circle P and intersects D on circle Q. Prove BE is parallel to FD. (Shown in figure)