13. I sent 10 candy bars by mail. Three melted. You walk over and randomly choose four candy bars. What is the probability that all of your candy bars were NOT melted? What is the probability that at least two of your candy bars were NOT melted?

ALL NOT MELTED $= \frac{7}{10} \cdot \frac{6}{9} \cdot \frac{5}{8} \cdot \frac{4}{7} = \boxed{\frac{1}{6}}$

AT LEAST 2 NOT melted $=$ $\underbrace{\text{2 not melted, 3 not melted, 4 not melted}}_{\text{must add these 3 probabilities}}$

NM = not melted
M = melted

ways to pick 4 bars:
NM, NM, NM, NM
M, NM, NM, NM
M, M, NM, NM
M, M, M, NM

recall

combinations $_nC_k = \frac{1}{k!} \cdot \frac{n!}{(n-k)!}$

k = how many you're choosing from "n"

3 permutations $_nP_k = \frac{n!}{(n-k)!}$

1) find combinations to create desired result

P(2 not melted) + P(3 not melted) + P(4 not melted)

↑ probability, not permutation

*order doesn't matter = combinations

2 NM

[combinations to choose 2NM] • [combos to choose 2 M]

$$\frac{7!}{2!(7-2)!} \cdot \frac{3!}{2!(3-2)!} = \frac{7!}{2! \cdot 5!} \cdot \frac{3!}{2!} = \frac{7 \cdot 6 \cdot 3}{2} \to 63$$

3NM

[combos to choose 3 NM] · [combos to choose 1 M]

$$\frac{7!}{3!(7-3)!} \cdot \frac{3!}{(3-1)!} = \frac{7!}{3! \cdot 4!} \cdot \frac{3!}{2!} = \frac{7 \cdot 6 \cdot 5}{2 \cdot 1} = 7 \cdot 3 \cdot 5 \to \underline{105}$$

4NM

[combos to choose 4 NM]

$$\frac{7!}{4!(7-4)!} = \frac{7!}{4! \cdot 3!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 7 \cdot 5 = \underline{35}$$

2) add. $63 + 105 + 35 = 203$

3) figure out how many possible combinations

choose 4 bars from 10 total

$${}_{10}C_4 = \frac{10!}{4!(10-4)!} = \frac{10!}{4!\cdot 6!} = \frac{10\cdot \cancel{9}^{3}\cdot \cancel{8}\cdot 7}{\cancel{4}\cdot \cancel{3}\cdot \cancel{2}} = 10\cdot 3\cdot 7 = 210$$

4) probability = $\frac{\text{combos to form desired result}}{\text{possible combos}}$

$$\boxed{\text{probability} = \frac{203}{210}}$$

ALT METHOD

Probability of at least 2 NM = 1 - probability of 1 NM

combos for 1 NM = [combos to choose 1 NM · combos to choose 3 M

$$\frac{7!}{1!(7-6)!} \cdot \frac{3!}{3!(3-3)!} = 7$$

total possible combos = 210

Probability of at least 2 NM = $1 - \frac{7}{210} = \boxed{\frac{203}{210}}$

14. Think about the group of numbers that are 7 digits long. For example, the number 1,215,389 is a number that is 7 digits long. Now, think of the group of numbers that are 7 digits long but have no "1" anywhere in the number. An example here would be the number 3,895,542. Does this latter group represent more than half of the former group? That, do 7-digit numbers having no "1" in their decimal representation constitute more than half of all 7-digit numbers?

all 7-digit #s $\underline{9}\cdot\underline{10}\cdot\underline{10}\cdot\underline{10}\cdot\underline{10}\cdot\underline{10}\cdot\underline{10} = 9\cdot 10^6$

7-digit #s w/ no 1 $\underline{8}\cdot\underline{9}\cdot\underline{9}\cdot\underline{9}\cdot\underline{9}\cdot\underline{9}\cdot\underline{9} = 8\cdot 9^6$

$$\frac{8\cdot 9^6}{9\cdot 10^6} = 0.47 \qquad \boxed{\text{NO}}$$

15. You are making a bag consisting of 20 pieces of candy to give to your cousin for Halloween and you can choose from 8 different types of candy. Assuming that you must give your cousin at least one of each type of candy, how many different bags can you make? *sticks + stones problem

* 20 stones = 20 pieces of candy

O O O O O O O O O O O O O O O O O O O O

*19 places to put a stick

*you have 7 sticks to ensure you get all 8 types of candy

$$\frac{19\cdot 18\cdot 17\cdot 16\cdot 15\cdot 14\cdot 13}{7!} = \boxed{50388}$$

← account for repeats

*could also do ${}_{19}C_7$

16. A class is made up of 10 girls and 7 boys. In how many ways can we make a 4-person debate team that has only boys? In how many ways can we make a 4-person debate team that has 2 boys and 2 girls? In how many ways can we make a 4-person debate team that has at least 1 girl? *combinations. order doesn't matter

only boys = choose 4 from 7 $= \frac{7!}{4!(7-4)!} = \frac{7!}{4!\cdot 3!} = \frac{7\cdot 6\cdot 5}{3\cdot 2} = 7\cdot 5 = \boxed{35}$

2 boys 2 girls = choose 2 from 7 · choose 2 from 10

$$= \frac{7!}{2!(7-2)!}\cdot\frac{10!}{2!(10-2)!} = \frac{7\cdot \cancel{6}^{3}}{\cancel{2}}\cdot\frac{\cancel{10}^{5}\cdot 9}{\cancel{2}\cdot 1} = 7\cdot 3\cdot 5\cdot 9 = \boxed{945}$$

at least 1 girl = 1 girl or 2 girls or 3 girls or 4 girls

*must add each of these

go to next page

1G [combos to get 1 girl · combos to get 3 boys

$$\frac{10!}{1!(10-1)!} \cdot \frac{7!}{3!(7-3)!} = 10 \cdot \frac{7 \cdot \cancel{6} \cdot 5}{3 \cdot 2} = 10 \cdot 7 \cdot 5 = 350$$

2G [combos to get 2 girls · combos to get 2 boys

$$\frac{10!}{2!(10-2)!} \cdot \frac{7!}{2!(7-2)!} = \frac{\overset{5}{\cancel{10}} \cdot 9}{\cancel{2}} \cdot \frac{7 \cdot \overset{3}{\cancel{6}}}{\cancel{2}} = 945$$

3G [combos to get 3 girls · combos to get 1 boy

$$\frac{10!}{3!(10-3)!} \cdot \frac{7!}{1!(7-1)!} = \frac{10 \cdot \overset{3}{\cancel{9}} \cdot \overset{4}{\cancel{8}}}{\cancel{3} \cdot \cancel{2}} \cdot 7 = 840$$

4G [combos to get 4 girls

$$\frac{10!}{4!(10-4)!} = \frac{10 \cdot \overset{3}{\cancel{9}} \cdot \cancel{8} \cdot 7}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2}} = 210$$

$$350 + 945 + 840 + 210 = \boxed{2345}$$

ALT. METHOD FOR LAST Q

ways to get at least 1 girl = total combinations of b+g − ways to get all boys

$$_{17}C_4 \quad - \quad _7C_4$$

$$\frac{17!}{4!(17-4)!} \quad - \quad \frac{7!}{4!(7-4)!}$$

$$\frac{17 \cdot \overset{4}{\cancel{16}} \cdot \overset{5}{\cancel{15}} \cdot \overset{7}{\cancel{14}}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2}}$$

↓

$$2380 \quad - \quad 35$$

↓

$$\boxed{2345}$$