Sets and Venn Diagrams Part 1
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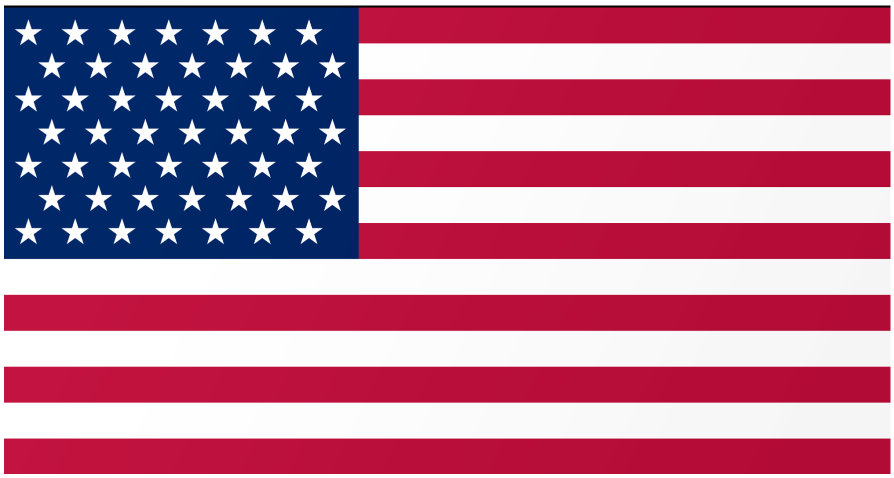
<u>Warm-up:</u>

What colors do we see on the American Flag?



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Today, we're going to be talking about ***sets***. A ***set*** is a clearly defined collection of distinct objects. Note that this is not really a definition. To define means to explain in simpler terms. Instead, all we do is replace one word, a ***set***, by another, a ***collection***. Plus, the meaning of the words to clearly define is not clearly defined. The problem is that the notion of a set is as fundamental as it is deep. It is impossible to explain it in simpler terms. The best we can do at the moment is to show a bunch of examples.

<u>Problem 1:</u> Let's start by considering the factors of **6**.

a. *What are these factors?*

   i. ______________________________

b. We can introduce notation to represent them, ***a set***. This will be written with *<u>curly braces</u>*, { and }. In a set, the <u>order doesn't matter</u>. Let's write the set of factors of 6 and call it S.

$S = \{$_____________________$\}$

c. Let's order these factors from smallest to largest. (Don't forget the curly braces)

$S_1 =$ _____________________

d. We can introduce another set of notation, an ***ordered set or a list***. This is given by *round brackets*, (and), or *parentheses*.

  i. *What do you think is the difference between a **set** and a **list**?*

  ____

  ii. Let's write the list of factors of 6 from least to greatest. (Don't forget the braces)

$L_1 =$ _____________________

e. Consider the two sets and two lists of letters:

$S_1 = \{C, A, T\}$ $S_2 = \{A, C, T\}$

$L_1 = (C, A, T)$ $L_2 = (A, C, T)$

  i. *Does $S_1 = S_2$? How do you know?*

  ____

  ii. *Does $L_1 = L_2$? How do you know?*

  ____

f. Returning to the warmup, if we let S_c be the set of colors of the American flag, what is in S_c ?

  ____

g. All the sets we've seen so far have something contained within it. What if a set does not contain any elements?

   i. *What do we call a set without any elements?*

      ____

   ii. We will represent the___________________ as ∅.

   iii. {A set of pigs that can fly by themselves.} = _______________

   iv. Give your own description and example of the empty set.

      ____

Notation:
∈ : The fact that the number 6 is an element of the set S_1 is denoted as $6 \in S_1$. The fact the 7 is not an element of the set S_1 is denoted as $7 \notin S_1$.

⊆ : For two sets S_1 and S_2, if every element of S_2 is also an element of S_1 ,then S_2 is a ***subset*** of S_1. In mathematical language, we write this as $S_2 \subseteq S_1$.

⊂: A subset of a set is called ***proper*** if it is not empty and is not equal to the original set.

Note: The notations ⊂ and ⊆ for sets are analogous to < and ≤ for numbers.

<u>Problem 2:</u> Some very special sets.

b. In the past, we've explored natural numbers. *What are natural numbers?*

   ____

c. We can put all of the natural numbers into a set and give it a special symbol: N.

   i. *Is 0 a part of the set of natural numbers? If not, how do you write it in mathematical language?*

      ____

ii. *How would you formally read your answer for (i)?*

____

iii. Let's represent N in set notation:

N = ____________________

d. We can also expand the set of natural numbers (N) to contain negative integers as well as zero. We will represent this set with the capital letter Z.

i. *Is 0 a part of the set of* ***integral numbers****, or just* ***integers****? How do you write it in mathematical language?*

____

ii. *How would you formally read your answer for (i)?*

____

iii. Let's represent Z in set notation:

Z = ____________________

e. We can further expand the set of ***integers***. Let $m \in Z$ and $n \in N$. The set Q of all the fractions m/n in the reduced form is called the set of ____________________.

i. Is $\frac{2}{3} \in Q$? _____________

ii. What about $\frac{4}{5}$? ___________

iii. Is the number 0.55 rational? Why or why not?

____

f. *Can we find all of the elements of N in Z? How can we use notation to compare these two sets?*

N ______ Z

i. *Is N a subset of N?* _______________

ii. *Is Z a subset of Z?* _______________

iii. *Is N a proper subset of Z?* If so, let's change our comparison to reflect this:

N ______ Z

g. Can we find all of the elements of Z in Q? How can we use notation to compare the two sets? *Is Z a proper subset of Q?*

Z ______ Q

<u>Problem 3:</u>

S_1= {C, A, T} S_2= {A, C, T}

a. *Is S_1 a subset of S_2? Is S_1 a proper subset of S_2?*

____

b. *Write down all the proper subsets of the set of colors of the US flag. Do they form a set? A list?*

____

c. Consider the set S_3 = { cat, horse, lion, tiger, 0, 5, 100, dog}. Find the following subsets:

i. Subset of numbers = __________________________

ii. Subset of animals = __________________________

iii. Subset of animals whose names contain the letter l = ____________________

iv. Subset of common pets = ________________________

v. Subset of colors = ___________________________

Red Hot Chilli Pepper

Move one digit to make the equality 101 - 102 = 1 correct

<u>Problem 4:</u> The number of elements in a set is called its ***cardinality***. In math language, we can write the cardinality of a set A as either |A| or card(A). We will use the former notation.

a. *Let U be the set of states in the United States. What is |U|?*

____

b. *What is |{0}|? Why?*

____

c. *What is |∅|?*

____

Next Time: Sets are a very important and useful idea. We can use this idea to mathematically explain a certain diagram we have previously explored: Venn Diagrams.

Challenge Questions

a. Decide which of the following two fractions is greater without cross-multiplying or bringing to the common denominator.

$$\frac{2017}{2018} \text{ or } \frac{2018}{2019}$$

b. Given the sets below, find which sets are subsets of another (Use the notation we have learned so far):
   i. A = set of all flowers
   ii. B = set of all red objects
   iii. C = set of all tulips
   iv. D = set of all balloons
   v. E = set of all things you can use for birthday decorations

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c. Consider the following sets:

A = {cat, horse, lion, tiger, 0, 5, 100, dog)

B = { 1 , 5, 100, panda, mouse, seahorse, lion}

   i. Can you make a new set that contains elements that are found both in A and B?

   ii. Can you make a new set that contains elements that are found in either A or B?

d. How many subsets, including the empty set and the set proper, does a set A of the following cardinality have?

   i. 2

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   ii. 3

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   iii. 4

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   iv. 5

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   v. Do you see a pattern?

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e. Given $|A| = n$, prove that the cardinality of the set of all the subsets of A is 2^n.