Combinations, Permutations, and Probability
after Spring Break

REVIEW

1. What is the difference between a combination and a permutation?

order doesn't matter — order matters

2. How many ways can you arrange all the letters in the word M-A-T-H?

$\frac{4 \cdot 3 \cdot 2 \cdot 1}{4!} = 24$

3. How many ways can you arrange two of the letters drawn from the word M-A-T-H?

$4 \cdot 3 = 12$

$\frac{4!}{2!} = \frac{4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}} = 4 \cdot 3$

4. Eight different runners are running in a race. The fastest three will win gold, silver, and bronze medals, respectively. How many different outcomes are possible?

$8 \cdot 7 \cdot 6 = 336$

$\frac{8!}{5!}$

5. Twenty basketball players form teams with 5 members each. How many different teams can be formed? order doesn't matter

$20 \cdot 19 \cdot 18 \cdot 17 \cdot 16$

$\frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{20!}{15! \cdot 5!}$

6. At a party, there are 30 people. If each person shakes hands with each other person, how many handshakes will occur?

$\frac{30 \cdot 29}{2}$ (person #1 · person #2) $= \frac{30!}{28! \cdot 2!} = 435$

NEW IDEA: PROBABILITY

Probability problems are often just counting problems. What is the probability of rolling the number "2" on normal 6-sided dice? Well, there are six possible outcomes, and only one of them is the desired one. Thus, the probability is 1 out of 6.

MIXED PRACTICE

7. The letters A-B-C-T-U-V are placed in a bag. You randomly draw three letters. What is the probability that you will draw the letters necessary to spell the word C-A-T?

total letters: 6

what we want / all choices

$\frac{3}{6} \cdot \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{20}$

(1st grab · 2nd grab · 3rd grab)

8. All of our names are in a hat, along with a bunch of other names, such that there are 100 names in total in the hat. I will draw four names at random. What is the probability that I draw the names Alex, Elili, Thea, and Olivia?

$\frac{4}{100} \cdot \frac{3}{99} \cdot \frac{2}{98} \cdot \frac{1}{97} = \frac{4!}{\frac{100!}{96!}} = \frac{4! \cdot 96!}{100!}$

9. There are 30 students in a class, including Ian and Sathvik. I choose two names at random to lead a discussion. What is the probability that I choose Ian and Sathvik?

$\frac{2}{30} \cdot \frac{1}{29} = \frac{1}{15 \cdot 29} = \frac{1}{435}$

$\frac{2!}{\frac{30!}{28!}} = \frac{2! \cdot 28!}{30!}$

10. How many distinct ways can all of the letters in the word G-E-O-M-E-T-R-Y be arranged to form new "words"?

$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 8!$

divide by 2

$\frac{8!}{2} = 20{,}160$

11. How many 4-digit numbers less than 7000 are odd, not even?

$6 \times 10 \times 10 \times 5 = 3000$

(1, 2, 3) (0, 1, 2) (0, 1, 2, 3) (0, 2, 4)

12. In how many ways can 5 people stand in a line? In how many ways can 5 people stand in a circle?

line: $5 \quad 4 \quad 3 \quad 2 \quad 1 = 5! = 120$

circle: answer will be different.
we will have less answers here

1 / 5, 2 / 4, 3 = 2 / 1, 3 / 5, 4

* divide by 5 $\frac{120}{5} = 24$

circle

13.