Lesson 2: Combinations and Pascal's Triangle

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Recall from the lecture that the Pascal's triangle is a way of writing numbers $\binom{n}{k}$ in a shape of a triangle where $\binom{n}{k}$ is the k-th number in the n-th row.

Problem 1.
What is the sum of all items in i's row of Pascal's triangle?

Problem 2.
a) Triangular number T_n is defined as $1+2+3+\cdots+n$. Find these numbers in Pascal's triangle.

b) Tetrahedral number S_n is defined as $T_1+T_2+T_3+\cdots+T_n$. Find these numbers in Pascal's triangle.

Problem 3.
Show that the k-th number in the n-th row of Pascal's triangle is equal to the number of ways to descend from the top of the triangle to the k-th position in the n-th row while moving down-right or down-left every time.

Problem 4.
Show that if $n \geq 4$ and $2 \leq k \leq n-2$, then

$$\binom{n}{k} = \binom{n-2}{k-2} + 2\binom{n-2}{k-1} + \binom{n-2}{k}$$

Problem 5.
If the ratios of three binomial coefficients are

$$\binom{n}{m+1} : \binom{n}{m} : \binom{n}{m-1} = 5:5:3$$

find n and m.

Problem 6.
Show that if p is prime and $1 \leq k < p$, then $p \mid \binom{p}{k}$.

Problem 7.
Let $ABCD$ be a cyclic quadrilateral, and let T be the intersection of lines AB and CD. Assume A lies on the segment TB and D lies on the segment TC. Show that $TA \cdot TB = TC \cdot TD$.

Problem 8.
Let BB_1 and CC_1 be altitudes in a triangle $\triangle ABC$. Show that the tangent line at A to the circumcircle of $\triangle ABC$ is parallel to B_1C_1.