## A LEVER PLAYING FIELD

MATH CIRCLE (BEGINNERS) 05/20/2012

You have a scale which has a rigid bar from which weights can be hung. The point at the center on which the scale balances is called the fulcrum. Use your intuition and experience to answer the following:

(1) If you have equal weights at equal distances from the fulcrum, the scale is:
(a) tilted down on the left
(b) balanced
(c) tilted down on the right

(2) If you have unequal weights at equal distances from the fulcrum, the scale is:
(a) tilted down on the side with the lighter weight
(b) balanced
(c) tilted down on the side with the heavier weight


(3) If you have equal weights at unequal distances from the fulcrum, the scale is
(a) tilted down on the side with the weight closer to the fulcrum
(b) balanced
(c) tilted down on the side with the weight further from the fulcrum


(4) The $5-\mathrm{lb}$. weights above are at equal distance from the cord their bar is hanging from (and that cord and the $10-\mathrm{lb}$. weight are at equal distances from the fulcrum). The scale will
(a) tilt down on the left
(b) balance
(c) tilt down on the right

We can actually hang weights directly from the top bar and it doesn't make a differenceso the following two pictures are equivalent:

(5) In these configurations, the balance
(a) tilts down on the left
(b) balances
(c) tilts down on the right
(6) What will happen if we remove the $5-\mathrm{lb}$. weight directly beneath the fulcrum?
nothing. weights $($ the fulcrum add no torque (force) in either direction for the system

(7) The balance above has 14 small, 1-lb. weights suspended evenly across the top bar. (The marks on top are 1-foot apart-they are just there to help you measure distances.) Suppose you wanted to replace the first 8 of them with a single large weight.
(a) How much should it weigh?
(b) Where should you place it? (How far from the fulcrum, on which side?)

Draw a picture of the new version, with the first 8 replaced. Label the distance and weight of the new weight:

(8) Now that you've replaced the first 8 small weights with one big one, you'd like to replace the last 6 of them with another single weight.
(a) How much should it weigh?
(b) Where should you place it?

Draw a picture of the new version, with both first 8 and last 6 replaced. Label the distances and weights of the two new weights:


Here's the 14 small weights again:

(9) Now I want to replace the first 4 small weights with one weight, and the last 8 small weights with a second large weight.
Draw a picture of the new version, labelling the distances and weights of the two new weights:

(10) Now replace the first 13 weights with a single weight, and leave the very last weight there.
Draw a picture of the new version, labelling the distances and weights of the two weights:

(11) In questions $7-10$, would it have been possible to give different answers than the ones you did?


8: no

The Law of the Lever, first proved by the Greek mathematician Archimedes, describes the situation in which a scale/balance/lever will be balanced.

Consider a lever that has a weight of $W_{1}$, at a distance $D_{1}$ to the left of the fulcrum, and a weight of $W_{2}$ at distance $D_{2}$ to the right of the fulcrum (as pictured). Then the Law of the Lever states that the two weights will be balanced if, and only if

$$
W_{1} D_{1}=W_{2} D_{2}
$$

More generally, if there are multiple weights on each side, then the sum of the weight $\times$ distance values on the left side, must equal the sum of the weight $\times$ distance values on the right side.

* diff color ballonstweights = diff possible

In the following pictures, your goal is to balance the beam. The jester weighs 1 unit of downward force, the weights have $1,2,3,4$, or 5 units of downward force, and the balloons weigh $1,2,3,4$, or 5 but they exert force upward.

For each picture, circle each balloon and weight which could be used to balance the beam if only that weight/balloon were attached to a single location on the bar that has a hook (note: hooks appear at distances of $1,2,3,4$, and 5 from the center on the left and right). Then draw at least one way that it can be done, using one of the weights/balloons you circled.
Downward force $=$ weights $(-)$


$=$


$=$

$=$
keno longer drawing where the objects go LAMC handout


$$
=
$$




## A LEVER PLAYING FIELD 2: NEVER SAY LEVER

MATH CIRCLE (BEGINNERS) 05/27/2012

The Law of the Lever, first proved by the Greek mathematician Archimedes, describes the situation in which a scale/balance/lever will be balanced.
Consider a lever that has a weight of $W_{1}$, at a distance $D_{1}$ to the left of the fulcrum, and a weight of $W_{2}$ at distance $D_{2}$ to the right of the fulcrum (as pictured). Then the Law of the Lever states that the two weights will be balanced if, and only if

$$
W_{1} D_{1}=W_{2} D_{2}
$$

More generally, if there are multiple weights on each side, then the sum of the weight $\times$ distance values on the left side, must equal the sum of the weight $\times$ distance values on the right side. For example if there were weights $W_{1}$ and $W_{2}$ on the left side at distances $D_{1}$ and $D_{2}$ respectively; and weight $W_{3}$ on the right side at distance $D_{3}$, then the scale will be balanced if and only if

$$
W_{1} D_{1}+W_{2} D_{2}=W_{3} D_{3}
$$

(1) You have a $3-\mathrm{lb}$. weight and a $5-\mathrm{lb}$. weight. On the lever below, place the weights so that the scale will be balanced. In this and all subsequent problems, label the weights you use (here $3 \mathrm{lb}, 5 \mathrm{lb}$ ), and indicate the distance of each one from the fulcrum. (Hint: There is not just one possible answer, although there is arguably one that is "simplest".)


[^0](2) You have a $1-\mathrm{lb}$. weight, a $2-\mathrm{lb}$. weight, and a 3-lb. weight. Place them on the scale so that it will be balanced, being sure to label the weights and their distances. (Again, there are multiple possible answers.)

(3) Place a $10-\mathrm{lb}$. weight on the following scale, so that it balances with a weight which is 15 ft . from the fulcrum on the right side. You choose the weight on the right side, and the distance of the $10-\mathrm{lb}$. weight.

\[

$$
\begin{aligned}
& 10 d_{1}=\omega_{2} \cdot 15 \\
& d_{1}=3 \mathrm{ft} \quad \omega_{2}=21 \mathrm{~b}
\end{aligned}
$$
\]

(4) Now solve problem (3) again, but give a different answer:


$$
\begin{aligned}
& 10 d_{1}=w_{2} 15 \\
& d_{1} 9 \mathrm{ft} \quad w_{2}=61 \mathrm{bs}
\end{aligned}
$$

(5) This scale will use a 3 kg weight, a 7 kg weight, and a 9 kg weight. The distances of the weights should be $2 \mathrm{~m}, 3 \mathrm{~m}$, and 3 m (but not necessarily in that order, and I'm not telling you on which side of the balance each distance should be!) Can you figure out where to place the weights, according to those rules, to balance the scale?


$$
\begin{aligned}
3(2)+7(3) & =9(3) \\
6+21 & =27
\end{aligned}
$$

(6) The following balance is not balanced. It currently has a weight of 2 kg suspended 1.6 m to the left of the fulcrum, and a weight of 1 kg suspended 6.4 m to the right. (Draw and label these weights and distances.) The total length of the bar is 20 m ( 10 on the left, 10 on the right). What is the weight of the smallest possible weight you could add to balance the scale, and where should you place it?


$$
3.2 \neq 6.4
$$

$t$ reed 3.2 units of fore on the left
put an 8 kg weight 0.4 meters to the left
(7) The following bar has a weight of 15 lbs . at the far left and and 3 lbs . at the far right (draw them!). Where along the bar should you place the fulcrum so that it will balance? (Draw and label!) You can imagine that the bar itself weighs nothing.

left of fulcrum $=x$
right of fulomm $=y$

$$
15 x=3 y
$$

entire bar $=L$

$$
\begin{aligned}
& x+y=L \\
& y=L-x-
\end{aligned}
$$

$$
\begin{aligned}
15 x & =3(L-x) \\
15 x & =3 L-3 x \\
18 x & =3 L \\
x & =\frac{1}{6} L ; y=\frac{5}{6} L
\end{aligned}
$$

(8) In the previous problem you were supposed to imagine that the bar weighed nothing. Of course in reality a bar WILL weigh something. The bar below is actually rather heavy; it weighs 4 lbs . The distance from the fulcrum to the left end of the bar is 2 feet, and I have hung a 1 lb . weight at the left end. There is nothing hanging on the right side.
How long is the bar? (Picture may not be to scale.) (Hint: Where is a bar's center of gravity, in general?) cuter of center of bar

(9) Archimedes reportedly once boasted about the usefulness of levers, saying, "Give me the place to stand, and I shall move the earth." (See picture-not to scale.)

(a) About how much mass ("weight") does the Earth have, in either lbs. or kilograms? (Use any knowledge you, your neighbors, your assistant, etc. have to formulate a best-guess estimate.)

(b) About how many lbs. or kilograms (use same as above) of downward force can a human generate? (Use any knowledge you, your neighbors, your assistant, etc. have to formulate a best-guess estimate.)

(c) Given your answers to (a) and (b), about how long a lever would Archimedes need in order to move (or just balance) the world, assuming he found a place to stand, and the fulcrum were placed 1 meter from where the world rested on the lever? (You can imagine the lever itself is weightless, like in Problem (7).)


## Extra Problems

1. The diagram below shows two rectangles whose sides are parallel to each other. By how much is the perimeter of the bigger rectangle greater than the perimeter of the smaller rectangle?

2. Classmates are standing in a circle. The teacher makes them do a headcount. Bianca says one, her neighbor says two, and so on. If they count in a clockwise direction, Antonia says five. If the count in a counterclockwise direction, Antonia says eight. How many students are forming the circle?


3. Two 1 cm long segments are marked on opposite sides of a square with side length 8 cm . The end points of the segments are connected with each other as shown in the diagram below. What is the area of the gray part?

4. Annie the ant starts at the left end of the stick and crawls $2 / 3$ of the length of the stick. Lenny the ladybug starts at the right end of the stick, upside down, and crawls $3 / 4$ of the length of the stick. How far apart are they from each other on the stick? Your answer should be in fraction form.

5. More than 800 people are taking part in a marathon. They either receive a blue shirt or a red shirt. $35 \%$ of the participants have a blue shirt. There are 252 more people with red shirts than blue shirts. How many people in total are participating in the marathon? 840

$$
\begin{aligned}
& \text { blue }=35 \% \\
& \text { red }=65 \%
\end{aligned}
$$

$$
\text { amount blue }+252=65 \%
$$

Total $=840$ participants

$$
252=30 \%
$$

6. The number sequence $2,3,6,8,8, \ldots$ is created by the following rule: The first two digits are 2 and 3 . After that, every subsequent digit is the unit digit of the product of the two previous digits. Which digit is the $2017^{\text {th }}$ element of the sequence?


Math Riddles

1. A duck gets $\$ 9$. A spider gets $\$ 36$. A bee gets $\$ 27$. How much does a dog get?

2. I am a number. Take away one thing and I become even. What number am I?
seven
take away"s", it becomes "ever"
3. I add 5 to 9 and I get 2 . How?
add 5 hours to 9:00, you get 2:00
4. Rearrange four 9 s to create an equation equal to 100 .

$$
99+\frac{9}{9}=100
$$

5. The day before yesterday, I was 21 . Next year, Ill be 24 . When is my birthday?

$$
\text { Dec } 31^{\circ}
$$

$\left(\begin{array}{l}\text { today is Jan } 1.2 \text { days ago you were 21. Now youre 22.) } \\ \text { this year youll be 23. Next year your 24! }\end{array}\right.$
6. In a certain pond, there is one Lilypad. Every day, the Lilypad doubles in size. By the $30^{\text {th }}$ day, the Lilypad covers the entire pond. How many days does it take to cover half of the pond?



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