PIGEON HOLE PRINCIPLE II

MATH CIRCLE (INTERMEDIATE) 11/20/2011

1) A city has a population of five million. It is known that no one has more than one million hairs on their head. Show that at least five people have the same number of hairs on their head.

2) Several football teams enter a tournament in which each team plays every other team exactly once. Show that at any moment in the tournament there will be two teams which have played, up to that moment, the same number of games.

3) Show that an equilateral triangle cannot be covered completely by two smaller equilateral triangles.

4) Fifty-one points are scattered inside a square with a side of 1 meter. Prove that some set of three of these points can be covered by a square with side 20 centimeters.

5) In a brigade of 7 people, the sum of the ages of the members is 332 years. Prove that three members can be chosen so that the sum of their ages is no less than 142 years.

6) On a certain planet in the solar system Tau Cetus, more than half the surface of the planet is dry land. Show that Tau Cetans can dig a tunnel straight through the center of their planet, beginning and ending on dry land. (Assume that their technology is sufficiently developed.)

7) Each box in a 3×3 arrangement of boxes is filled with one of the numbers $-1, 0, 1$. Prove that of the possible sums along the rows, the columns, and the diagonals, two sums must be equal.

8) Fifteen boys gathered 100 nuts. Prove that some pair of boys gathered an identical number of nuts.

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg "Mathematical Circles (Russian Experience)"
- Previous UCLA Math Circle notes