An Introduction to Polyhedra

UCLA Olga Radko Math Circle Beginners 2

2/21/2021

Warm-Up: Fraction Clinic

Question 1:

$\frac{2}{3} - \frac{1}{3}$ = 1/3

$\frac{2}{3} - \frac{1}{2}$ = 1/6

$\frac{2}{3} - \frac{1}{4}$ = 5/12

$\frac{2}{3} - \frac{1}{5}$ = 7/15

$\frac{2}{3}$ x $3 \div 5$ = 2/5

$\frac{2}{3}$ $\div 5$ x 3 = 2/5

$\frac{2}{3}$ x $\frac{3}{5}$ = 2/5

$\frac{2}{3}$ $\div$ $\frac{5}{3}$ = 2/5

<u>Question 2:</u> *Put the correct sign, > , <, or =, in between the numbers without bringing fractions to the common denominator and without guessing.*

$$\frac{15}{16} < \frac{16}{17}$$

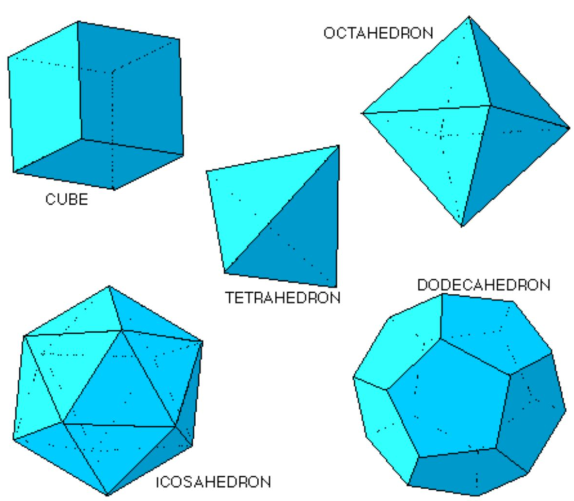
$$\frac{7}{8} < \frac{15}{17}$$

$$\frac{8}{3} > \frac{5}{2}$$

This week, we'll dive into the world of polyhedra!

Lesson

<u>Problem 1:</u> Below are some examples of polyhedra:



a. *Based on the examples above, what are some characteristics you notice about polyhedra?*

Answers may vary (3-D; Symmetries; Made with familiar shapes like squares, triangles, pentagons)

b. In order to make a polyhedron we need three things:

   i. *Vertex: A point which is at the corner of a polyhedron*

   ii. *Edge: A line segment that connects two Vertices.*

   iii. *Face: A polygon that is bounded by several Edges of the polyhedron.*

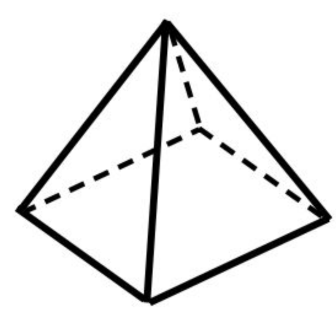
   iv. *What is the smallest number of vertices and edges you need to make a face? Draw the shape. **Triangle***

3 Edges and 3 Vertices

c. Based on what we've seen so far, how can we define a polyhedron?

   i. *A polyhedron is a 3-dimensional shape made up of several vertices, straight edges, and faces.*

Problem 2: Answer the questions below about the following polyhedron:



a. *How many **vertices** does the polyhedron have?*

5

b. *How many **edges** does the polyhedron have?*

8

c. *How many **faces** does the polyhedron have?*

5

Problem 3:

a. *Can a polyhedron have three vertices? Why or why not?*

No, we need a minimum of three vertices to create a face, but a polyhedron needs to have multiple faces.

b. *What is the smallest number of vertices that a polyhedron can have?*

4 (Tetrahedron)

c. *What is the smallest number of edges a polyhedron can have? Why is it that number and not another?*

We need at least 5 edges to make a polyhedron. Like for any number of vertices, we need at least 3 edges to make a face, but a tetrahedron is the smallest polyhedron which has 6 edges.

d. *What is the smallest number of faces a polyhedron can have?*

4 (Tetrahedron)

Red Chilli Pepper Problem

James thinks of a number such that the sum of one third of the number and one fourth of the number equals twenty one. What number does James think of?

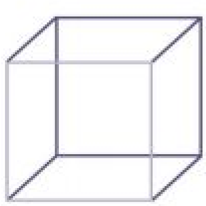
Numbers divisible by both 3 and 4: 12, 24, 36 …

36 works!

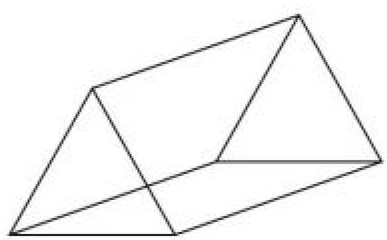
Problem 4: Fill in the following table and use the space below to draw the shapes.

#	Polyhedron	Vertices	Edges	Faces
1	Cube	*8*	*12*	*6*
2	Triangular Prism	*6*	*9*	*5*
3	5-Prism	*10*	*15*	*7*
4	Pyramid	*5*	*8*	*5*
5	Tetrahedron	*4*	*6*	*4*
6	Octahedron	*6*	*12*	*8*
7	“Tower”	*9*	*16*	*9*
8	Cube with a Cut Corner	*10*	*15*	*7*
9	(Optional) Your Own			

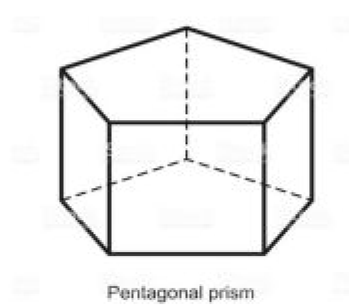
Cube



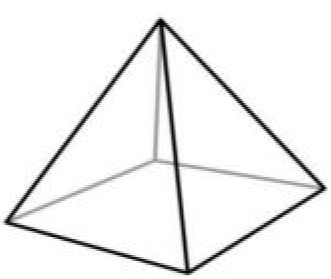
Triangular Prism



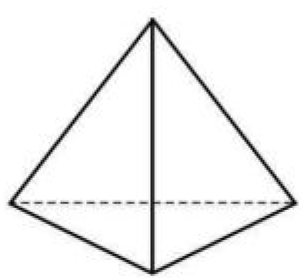
5-Prism



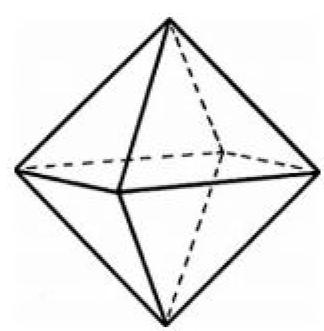
Pyramid



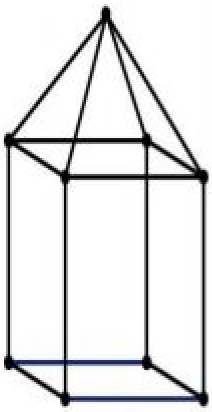
Tetrahedron

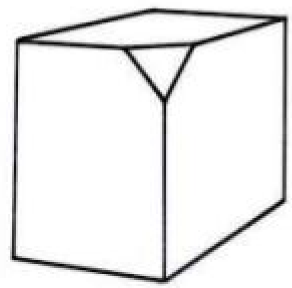


Octahedron



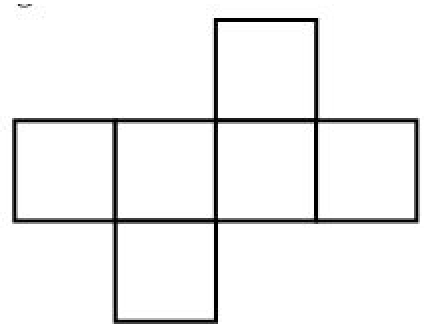
Tower | *Cube with a Cut Corner* | *(Your Own)*

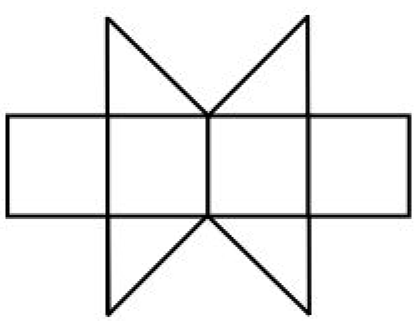


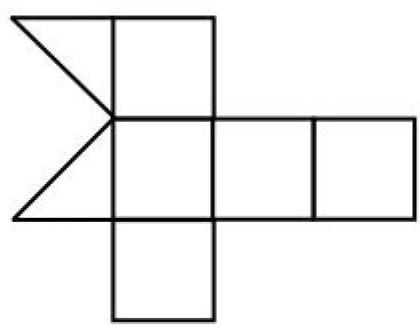


Problem 5: *Which of the following nets can be folded into a cube? Circle the correct ones:*

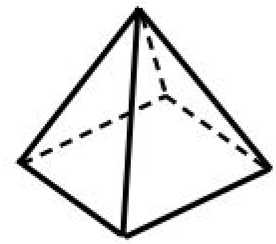
These work!







Problem 6: Pyramids



a. A pyramid is a type of polyhedra that has the following properties:
   i. The base is a polygon
   ii. All the vertices of the *base* are connected with a special vertex called an apex.
   iii. Circle the apex of the pyramid to the right.

b. *How many **vertices**, **edges**, and **faces** does the pyramid in the diagram above have?*
   i. 5 Vertices
   ii. 8 Edges
   iii. 5 Faces

c. *What about a pyramid with 8 **vertices**? Draw the pyramid in the space below.*
   i. 14 Edges
   ii. 8 Faces

d. *What about a pyramid with 10 **vertices**? Draw the pyramid in the space below.*
   i. 18 Edges
   ii. 10 Faces

e. *What about a pyramid with F **faces**? (Hint: Think about how the number of edges relates to the number of vertices in the polygon base).*
   i. 2F-2 Edges
   ii. F Vertices

f. *Is it possible for a pyramid to have 2021 **vertices**?*

   Yes; 2021 Faces, 4040 Edges

g. *Is it possible for a pyramid to have 2021 **edges**?*

   No, the number of edges must be even. (2023/2) = number of vertices/faces.

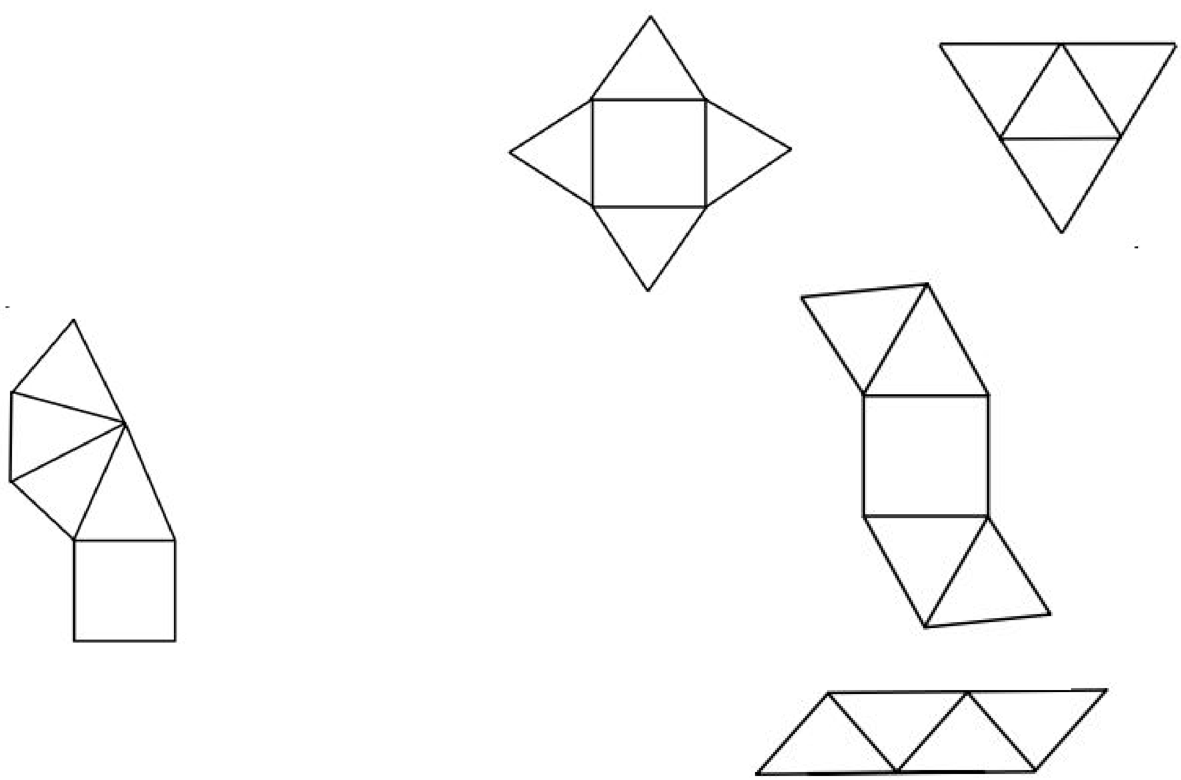
Something to Consider: We've found a neat relationship between faces, edges, and vertices for Pyramids. Does a similar relationship exist for all Polyhedra? How will this new relationship compare to the relationship we found for Pyramids? We'll explore these questions in the second part of this topic.

Challenge Questions

Question 1: Tyler had five friends at his birthday party. He gave his first friend one sixth of the birthday cake. The second kid got one fifth of what was left. The third friend got one fourth of the remains, and the fourth got one third of what was left after that. Finally, Tyler took a half of what was left of the cake for himself and gave the other half to his fifth friend. Is this a fair way to divide a cake between the friends? If not, who got the biggest part of the cake?

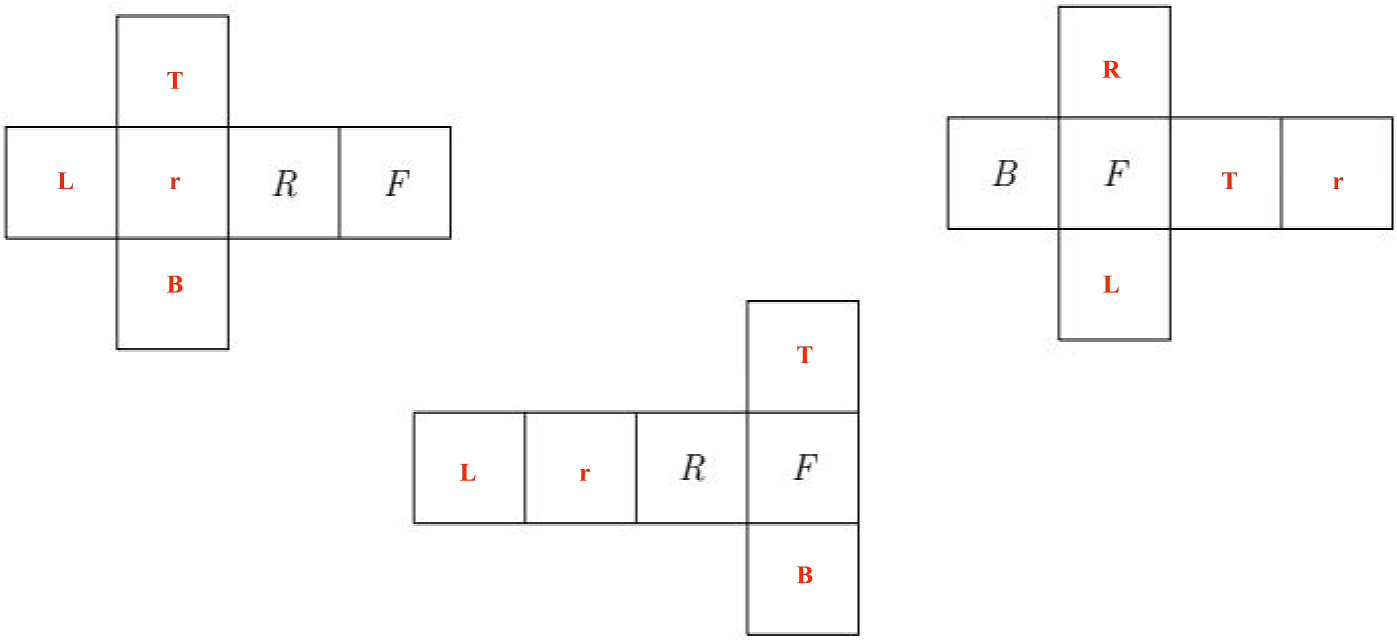
Yes, cut a cake into 6 pieces. If A takes 1/6th, 5 pieces remain. If B takes 1/5th of the remaining, they take 1 piece, leaving 4. If C takes ¼ of the remaining, they take 1 piece leaving three. Each of the pieces are ⅙ so each person gets ⅙ of the whole cake.

Question 2: Circle the following nets that can be folded into a pyramid: *These nets work!*



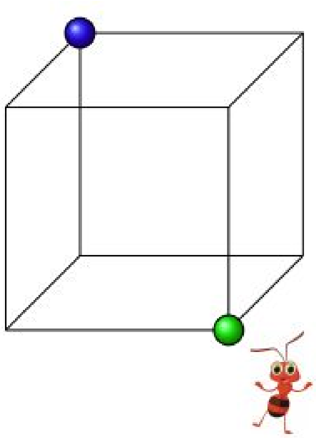
Question 3: Recall that for a net of a cube, the letter **B** denotes the *bottom* face of the cube, **L** stands for *left*, **R** stands for *right*, **T** for *top*, **F** for *front*, and the lower-case **r** stands for *rear*.

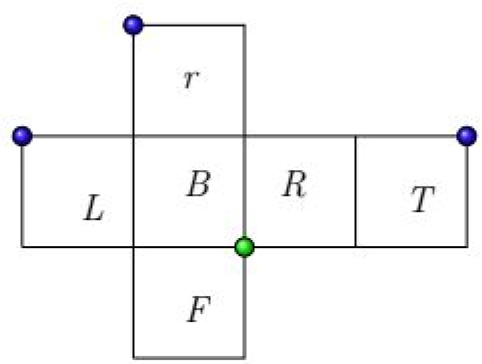
Complete labeling the following nets:



Question 4: An ant needs to find the shortest possible path from a lower corner of a cubic room to the opposite upper corner. The corners are shown as a green and blue dot on the picture below. The ant can crawl on the floors, walls, and ceiling of the room, but it cannot fly through the air. *Draw at least three different shortest paths on the picture.*

Use the net of the cube provided below. (Hint: also use the symmetries of the cube)





The ORMC Logo

Above is a picture of the ORMC logo.

1. What are some characteristics you notice about the shape within the logo?

   Answers will vary

   a. What dimensional shape is it? *4-dimensional.*

Looking at the picture in the center of the logo, an untrained eye may spot a large number of symmetries that make the picture intriguing. A trained eye will see eight 3-dimensional cubes forming faces of a **4-dimensional cube**, also known as a **hypercube** and a **tesseract**.

Let's compare drawing a 3D cube and a 4D cube on a 2D sheet of paper. This shape in the lower-right corner below is the tesseract depicted in the logo.