1. Find all possible values of $k$ such that

\[
\frac{(2k)!}{(2k-1)!} = \frac{(k+6)!}{(k+5)!}
\]

\[
\frac{(2k) \cdot (2k-1) \cdots 2k}{(2k-1) \cdots 2k} \quad \frac{(k+6) \cdot (k+5) \cdot (k+4) \cdots}{(k+5) \cdot (k+4) \cdots}
\]

$2k = k + 6 \Rightarrow 2k - k = 6 \Rightarrow k = 6$

2. Simplify

\[
\frac{(x^2 - 4)!}{(x-2) \cdot (x^2 - 5)!}
\]

\[
\frac{(x^2 - 4)(x^2 - 5)(x^2 - 6) \cdots}{(x-2)(x^5)(x^6) \cdots}
\]

\[
\frac{x^2 - 4}{x-2} \quad \frac{(x+2)(x-2)}{(x-2)}
\]

\[
\frac{x+2}{x+2}
\]

3. How many 5-digit numbers can be formed with only odd numbered digits? How many of these numbers are bigger than 70,000?

\[
\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5} = 5^5 = 3125
\]

bigger than 70,000 $\Rightarrow \frac{7 \text{ or } 9}{2 \cdot 5 \cdot 5 \cdot 5 \cdot 5} = 2 \cdot 5^4 = 1250$
(g) Simplify
\[
\frac{17! \cdot 0!}{14! \cdot 3!} = \frac{17 \cdot 16 \cdot 15 \cdot 14!}{2 \cdot 1} = 17 \cdot 8 \cdot 5
\]

(h) Compare \(\frac{(n+1)!}{(n-1)!}\) with \(n^2\) for any positive integer \(n\):
\[
\frac{(n+1)!}{(n-1)!} = \frac{(n+1) \cdot n \cdot (n-1) \cdot (n-2) \cdots}{(n-1) \cdot (n-2) \cdots} = (n+1) \cdot n = n^2 + n
\]
\[
\therefore \frac{(n+1)!}{(n-1)!} > n^2
\]

2. Find all possible values of \(k\) such that
\[
\frac{(2k)!}{(2k-1)!} = \frac{(k+6)!}{(k+5)!}
\]
\[
\frac{(2k)!}{(2k-1)!} = 2k
\]
\[
\frac{(k+6)!}{(k+5)!} = k+6
\]
\[
2k = k + 6 \quad \therefore k = 6
\]

3. (a) Last Sunday, at the end of class, all 29 students in the Intermediate group were to form a line at the front of the classroom. In how many ways could they have formed this line?
\[
29!
\]

(b) At every meeting, Jeff selects 10 students (out of 29) from the class to form the line. He wants to have a different line every time. How many meetings in a row can they continuing forming new lines each time?
\[
\frac{29!}{19!}
\]
Additional Combinatorics Problems

1. There are 10 students in a club. How many ways are there to pick a President and Vice President from the club’s members?

\[ 10 \cdot 9 = 90 \]

2. There are 10 students in another club. How many ways are there to select a committee of two people from the club’s members?

\[ \frac{10 \cdot 9}{2} = 45 \]

3. How many ways are there to choose two items out of \( n \) choices, if the order matters?

\[ (n) \cdot (n-1) = \frac{n!}{(n-2)!} \]

4. How many ways are there to choose two items out of \( n \) choices, if the order does not matter?

\[ \frac{n \cdot (n-1)}{2} = \frac{1}{2} \cdot \frac{n!}{(n-2)!} \]

5. How many ways are there to choose three items out of \( n \) choices, if the order matters?

\[ n \cdot (n-1) \cdot (n-2) = \frac{n!}{(n-3)!} \]

6. How many ways are there to choose three items out of \( n \) choices, if the order does not matter?

\[ \frac{n \cdot (n-1) \cdot (n-2)}{3!} = \frac{1}{3!} \cdot \frac{n!}{(n-3)!} \]
How many ways are there to choose \( k \) items out of \( n \) choices?

a) If order does matter?

\[
\frac{n!}{(n-k)!}
\]

Choose \( k \) items from \( n \)

b) If order doesn't matter?

\[
\frac{1}{\text{duplicates}} \cdot \frac{n!}{(n-k)!}
\]

\[
\frac{1}{k!} \cdot \frac{n!}{(n-k)!}
\]
VOCABULARY ALERT

**combinations** involve situations where order does not matter

**permutations** involve situations where order does matter

**serendipity** is a fancy word that means “good fortune!”

7. There are 20 students in a gym class. How many ways are there to pick a basketball team consisting of 5 people? Is this a combinations problem, or a permutations problem? Did you get the answer right by calculating it correctly, or through serendipity?

![Combinations calculation]

8. HARD QUESTION. There are six letters in the Swiftian language. A word is any sequence of six letters, some pair of which are the same. (The identical letters do not have to be next to each other in the word). How many words are there in this wonderful language?

9. Do 7-digit numbers having no digits 1 in their decimal representation constitute more than half of all 7-digit numbers?

10. You are making a bag consisting of 20 pieces of candy to give to your cousin for Halloween and can choose from 8 different types of candy. Assuming you give your cousin at least one of each types of candy, how many different bags could you make?
# 8: Swiftian Language

*there are many ways to complete this problem, this is just one of them!*

Step 1: assume repeat letter is last letter

\[ \underline{6} \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot \underline{5} \]  
\[ 6! \cdot 5 \]

Step 2: account for positions of repeat letter.

repeat letter can go in 6 different spots.  
\[ 6! \cdot 5 \cdot 4 \]

Step 3: account for duplicate words

For example, if you are repeating the letter a, your repeat letter being in position 3 vs being in position 5 gives you the same word... because same letter?

\[ bcaade = bacade \]

So we've double counted:

\[ \frac{6! \cdot 5 \cdot 4}{2} = 10800 \]