

# Homework ☺

1. What's the sum of the first 10 terms? \* stay in fraction form

$$\left[ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \dots \right]$$

$$r = \frac{1}{2}$$

$$S_{10} = \frac{\frac{1}{2} (1 - (\frac{1}{2})^{10})}{1 - \frac{1}{2}} = \frac{\frac{1}{2} (1 - \frac{1}{1024})}{\frac{1}{2}} = \frac{1024}{1024} - \frac{1}{1024} = \frac{1023}{1024}$$

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

2. What's the sum of the first 5 terms?

3. Write in series notation ( $\sum_{?}^{?}$ )  
 $7 + 35 + 175 + 875 + \dots + 21875$

$$\sum_{n=1}^6 7 \cdot 5^{(n-1)} \quad \text{OR} \quad \sum_{n=0}^5 7 \cdot 5^n$$

4. Write in series notation ( $\sum_{?}^{?}$ )

$$\frac{1}{18} + \frac{1}{36} + \frac{1}{72} + \frac{1}{144} + \frac{1}{288} + \frac{1}{576}$$

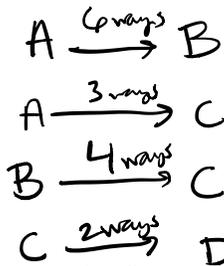
$$\sum_{n=1}^6 \frac{1}{18} \cdot \frac{1}{2^{(n-1)}} \quad \text{OR} \quad \sum_{n=0}^5 \frac{1}{18} \cdot \frac{1}{2^n}$$

# Combinatorics

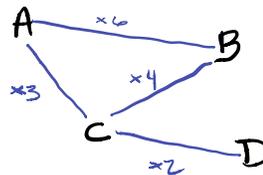
- Suppose there are 5 different teacups, 3 different tea saucers, and 4 different teaspoons in the "Tea Party" store. How many ways are there to buy a set consisting of a cup, a saucer, and a spoon?

$$5 \cdot 3 \cdot 4 = \boxed{60}$$

- There are four towns A, B, C, and D in Wonderland. In Wonderland, all roads are one-way. Suppose there are 6 different roads from A to B, 3 from A to C, 4 B to C, and 2 C to D. How many ways are there to get from A to C?



$$3 + 6 \times 4 = 27$$



- Re-visit the "Tea Party" store from problem 1. How many ways are there to buy two different items from the set? For example, we could buy one of the cups and one of the spoons, but not two spoons.

• 5 teacups (T)  $\begin{matrix} (T+Sa) \\ 5 \times 3 \end{matrix}$  +  $\begin{matrix} (T+Sp) \\ 5 \times 4 \end{matrix}$  +  $\begin{matrix} (Sa+Sp) \\ 3 \times 4 \end{matrix}$   
 • 3 saucers (Sa)  $15 + 20 + 12$   
 • 4 spoons (Sp)

$$\boxed{47}$$

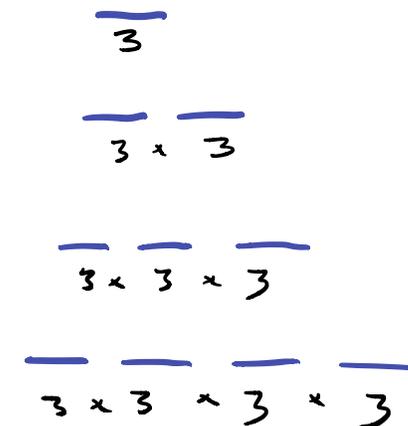
- This time, we have an alphabet with only the letters 'A', 'B', and 'C'. A word in this language is an arbitrary sequence of no more than 4 letters. How many words does the language contain?

A, B, C

$$3 + 3^2 + 3^3 + 3^4$$

$$3 + 9 + 27 + 81$$

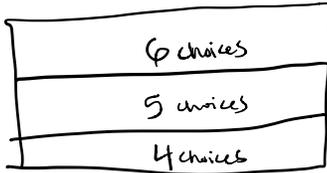
$$\boxed{120}$$



A B C D E F

5. We have six pieces of fabric, each a different color. How many ways are there to sew a flag with 3 horizontal strips (each of a different color) of equal height? Assume that we can distinguish the top of the flag from the bottom. (What if that were not true?)

• 6 color choices



$$6 \cdot 5 \cdot 4 = \boxed{120}$$

\*SO Red+White+Blue  $\neq$  Blue+White+Red

if it were not true,

R+W+B flag = B+W+R flag.. so we double counted!

$$120 \div 2 = \boxed{60}$$

6. Same situation, but now we have 200 colors and 40 stripes. What would the math look like in that version of the problem? Do not solve. We like you too much to ask you to actually solve this one to a number. (If we are going to continue doing problems like these, it would be great to have some convenient way to write down patterns of multiplications, wouldn't it?)

$$200 \times 199 \times 198 \times 197 \times \dots \times 161$$

If  $n$  is a natural number, then  $n!$  (pronounced “ $n$  factorial”) is the product

$$n! = n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot 3 \cdot 2 \cdot 1.$$

For example,  $2! = 2$ ;  $3! = 6$ , and  $4! = 24$ . We also define  $0! = 1$ .

1. Simplify the following expressions:

(a)

$$5! \cdot 6 \cdot 7 = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 6 \cdot 7 = 7!$$

(b) Express your answer using a factorial:

$$2 \cdot 3 \cdot 4 = 4!$$

(c)

$$\frac{27!}{25!} = 27 \cdot 26$$

(d) Write this product as a ratio of two factorials:

$$n \cdot (n - 1) = \frac{n!}{(n-2)!}$$

(e) Simplify the fraction:

$$\frac{n!}{(n-3)!} = n \cdot (n-1) \cdot (n-2)$$

(f) Simplify the fraction

$$\frac{(2n-1)!}{(2n-3)!} = (2n-1)(2n-2)$$

(g) Simplify

$$\frac{17! \cdot 0!}{14! \cdot 3!} = \frac{17 \cdot \overset{8}{\cancel{16}} \cdot \overset{5}{\cancel{15}}}{3 \cdot 2 \cdot 1} = 17 \cdot 8 \cdot 5$$

(h) Compare  $\frac{(n+1)!}{(n-1)!}$  with  $n^2$  for any positive integer  $n$ :

$$\frac{(n+1)!}{(n-1)!} = \frac{(n+1) \cdot n \cdot (n-1) \cdot (n-2) \dots}{(n-1) \cdot (n-2) \dots} = (n+1) \cdot n = n^2 + n$$

$\therefore \frac{(n+1)!}{(n-1)!} > n^2$

2. Find all possible values of  $k$  such that

$$\frac{(2k)!}{(2k-1)!} = \frac{(k+6)!}{(k+5)!}$$

3. (a) Last Sunday, at the end of class, all 29 students in the Intermediate group were to form a line at the front of the classroom. In how many ways could they have formed this line?

(b) At every meeting, Jeff selects 10 students (out of 29) from the class to form the line. He wants to have a different line every time. How many meetings in a row can they continue forming new lines each time?