

Extra Problems

week 9

adapted from *Mathematical Circles*
Fomin, Genkin, Itenberg

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Problem 4.

Prince Ivan has two magic swords to fight a hydra. One can cut off 21 heads at once, while the other can cut off 4 heads at once. Also, after the second sword cuts off heads, the hydra grows 1985 new heads. Can Prince Ivan cut off all the heads of the hydra if originally it has 100 heads?

Problem 5.

There are three printing machines, the first accepts a card with two numbers a and b ; then returns a card with the numbers $a + 1$ and $b + 1$. The second takes in a card with two even numbers, a and b ; then returns a card with the numbers $\frac{a}{2}$ and $\frac{b}{2}$. The third accepts two cards with the numbers a and b on the first, and b and c on the second. The machine then returns a card with the numbers a and c . All these machines also return the cards originally put into them. If originally you only have a card with the numbers 5 and 19 is it possible to obtain a card with the numbers 1 and 1988?

Problem 6.

A pawn moves across a $n \times n$ chessboard such that it can move either one square to the right, one square upward, or along a diagonal one square down-left. Can the pawn go through all the squares on the board visiting each square exactly once, and finishing its trip on the square to the right of the the starting square?