

Sequences & Series

* Last time, we learned about arithmetic sequences

- ① a sequence is an ordered list of numbers

$$\underbrace{[2, 4, 6, 8]}_{k=2}$$

- ② a series is the sum of the terms in a sequence

$$2+4+6+8 = \underline{20}$$

* more on series later!

- ③ each element in a sequence can be numbered + named.

$$a_0 = 0$$

$$a_1 = 2$$

$$a_2 = 4$$

$$a_3 = 6$$

$$a_4 = 8$$

$$a_n = \underline{2 \cdot n}$$

$$\begin{aligned} & [a_0, a_1, a_2, a_3, a_4, \dots, a_n] \\ & [0, 2, 4, 6, 8, \dots, \underline{2n}] \end{aligned}$$

* Sequences can be defined 2 ways. What types of definitions are these?

$$a_n = 2n + 1 \text{ explicit definition}$$

$$a_{n+1} = a_n + 2 \text{ recursive definition}$$

Refresher on Arithmetic Sequences

1. First term = 4, 10th term = 67. What's the common difference?

$$67 - 4 = 63$$

we make 9 "jumps" from 1st term to 10th term

$$63 \div 9 = 7 \quad k=7$$

2. 5th term = 11. 10th term = 41. What's the first term?

5 "jumps" from 5th term to 11th term

$$41 - 11 = 30 \quad 30 \div 5 = 6 \quad k=6$$

$\boxed{-13}$ $\begin{array}{ccccccc} -3 & -1 & 1 & 5 & 11 \\ \uparrow^k & \uparrow^k & \uparrow^k & \uparrow^k & \uparrow^k \end{array}$

3. [3, 7, 11, 15 ...]; what's the 50th term?

$$k=4$$

49 jumps from 1st term to 50th term

$$3 + 49 \cdot 4 = \boxed{199}$$

Series

* we use Σ to denote a series

1. $\sum_{n=1}^{10} a_n$; $a_1 = 2$; $k=2$ RAINBOW Method!

write the sequence, then add

2, 4, 6, 8, 10, 12, 14, 16, 18, 20

$$22 \times 5 = \boxed{110}$$

$$2. \sum_{n=1}^8 a_n = ? \quad a_n = 2n+2$$

4, 6, 8, 10, 12, 14, 16, 18
22

$$22 \times 4 = 88$$

$$3. \sum_{n=1}^{50} a_n = ? \quad a_n = 3n - 1$$

$a_1 = 2$
 $a_2 = 5$
 $a_3 = 8$
 \vdots
 $a_{49} = 146$
 $a_{50} = 149$

\uparrow
 2
 $3 \cdot 3 - 1$

$$151 \times 25 = \boxed{3775}$$

$$4. \sum_{n=1}^{51} a_n = ? \quad a_n = 3n - 1$$

$$3775 + (149 + 3)$$

$$3775 + 152 = \boxed{3927}$$

$$5. \sum_{n=1}^5 3^n = ?$$

$3^1, 3^2, 3^3, 3^4, 3^5$
 \downarrow
 $9 \cdot 3$
 $27 \cdot 3$
 $3, 9, 27, 81, 243$

$$\boxed{363}$$

*can't rainbow!

$$\sum_{n=1}^{\infty} a_n = ?$$

*Can we rainbow? kinda!

$$a_n = 2n+2$$

*add #s on end then
divide by 2 to
find middle term!
 $2+24=28$
 $28 \div 2 = 14$

$$28 \times 5 + 14 = \boxed{154}$$



Summing Arithmetic Sequences

*recall: arithmetic sequences follow this form

$$[a_0, a_0+k, a_0+2k, a_0+3k \dots]$$

$\downarrow k$ $\downarrow k$ $\downarrow k$

*common difference between each term

sum of first
2 terms

$$S_2 = a_0 + [a_0+k] = 2a_0 + k$$

$$S_3 = a_0 + [a_0+k] + [a_0+2k] = 3a_0 + 3k$$

$$S_4 = a_0 + [a_0+k] + [a_0+2k] + [a_0+3k] = 4a_0 + 6k$$

*how can we generalize S for
n terms?

$$S_n$$

$$S_n = \underbrace{[a_0]}_{-} + \underbrace{[a_0+k]}_{-} + \underbrace{[a_0+2k]}_{-} + \dots + \underbrace{[a_0+(n-1)k]}_{-}$$

$$= \underbrace{[na_0]}_{-} + \underbrace{[k+k+k+k+\dots+k]}_{(n-1)k}$$

factor out k, we're left with...

*cool formula!

$$\boxed{1+2+3+\dots+n = \frac{n(n+1)}{2}}$$

example

$$1+2+3+4+5$$

$$\frac{5 \times 6}{2} = \frac{30}{2} = 15$$

$$1+2+3+4+\dots+(n-1)$$

$$= \frac{(n-1)(n-1+1)}{2}$$

$$\boxed{\frac{(n-1)n}{2}}$$

$$= \underbrace{[na_0]}_{-} + k \underbrace{[1+2+3+4+\dots+(n-1)]}_{-}$$

$$\boxed{S_n = na_0 + \frac{k \times (n-1)(n)}{2}}$$

Try Sum! (no pun intended)

1. $[1, 5, 9, 13, \dots]$; What's the sum of the first 16 terms?

$$n=16 \quad S_n = S_{16} = 16 \cdot 1 + \frac{4 \cdot 15 \cdot 16}{2}^8$$

$$a_0 = 1 \quad = 16 + 4 \cdot 15 \cdot 8 = 496$$

$$k = 4 \quad \begin{matrix} 60 \cdot 8 \\ 480 \end{matrix}$$

2. Sum of the first 30 terms? $[50, 45, 40, 35, \dots]$

$\begin{matrix} \nearrow & \nearrow & \nearrow \\ -5 & -5 & -5 \end{matrix}$

$$S_n = n a_0 + \frac{k(n-1)n}{2}$$

$$a_0 = 50$$

$$n = 30$$

$$k = -5$$

$$S_{30} = 30 \cdot 50 + \frac{-5(29)(30)}{2}$$

$$S_{30} = -675$$