

DIVISIBILITY II

MATH CIRCLE (INTERMEDIATE) 11/06/2011

1) The number x is not divisible by 3. Is it possible that the number $2x$ is divisible by 3?

2) The number x is even. Is it true that $3x$ must be divisible by 6?

3) The number $5x$ is divisible by 3. Is it true that x must be divisibly by 3?

4) The number $15x$ is divisible by 6. Is it true that x must be divisible by 6?

5) Prove that the product of any five consecutive natural numbers is divisible by 30. Is the product of five consecutive natural numbers divisible by 120?

6) Suppose p is a prime. How many natural numbers are a) less than p and relatively prime to p ? b) less than p^2 and relatively prime to p ?

7) How many zeros are there at the end of $100!$?

Challenge 1) The numbers a and b satisfy the equation $56a = 65b$. Prove that $a + b$ is composite.

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg “Mathematical Circles (Russian Experience)”
- Previous UCLA Math Circle notes