Homework Review

\[ x = 0.8\overline{23} \]
\[ 10x = 8.\overline{23} \]
\[ 1000x = 823.\overline{23} \]
\[ 1000x - 10x = 990x \]
\[ 823.\overline{23} - 8.\overline{23} = 815 \]

\[ x = \frac{815}{990} \]

\[ 0.12\overline{32} \]

\[ x = 0.9\overline{9} \]
\[ 100x = 99.\overline{9} \]
\[ 100x - x = 99 \]
\[ 99.\overline{9} - 0.\overline{9} = 99 \]
\[ 99x = 99 \]
\[ x = \frac{99}{99} = 1 \]

2. Is there a difference? 0.722\overline{4} vs. 0.722\overline{4}2

\[ 0.72242424\ldots \text{ same!} \]

3. Is this stuff useful?

maybe...
Sequences

1. What is a rational number?  
a number you can write as a fraction

2. What are some examples of irrational numbers?  
$\pi, \sqrt{2}, \sqrt{3}$

3. Let’s look at some sequences.  
$\langle 0, 2, 4, 6, 8, 10, \ldots \rangle$

$n =$ 0, 1, 2, 3, 4, 5, ...

What’s the pattern? +2 each time

even numbers

explicit definition  
$a_0 = 2 \cdot n$

$\quad a_{15} = 30$

$n = 100 \rightarrow 200$

recursive definition  
$q_{n+1} = a_n + 2$

\[ q_{n+1} = \frac{a_n + 2}{10 + 2} \]

$n = 5$

$\quad a_{n+1} = a_6 \rightarrow 12$

$\quad a_8 \rightarrow 14$

4. Write the first 6 perfect squares.  
$\langle 1, 4, 9, 16, 25, 36, \ldots \rangle$

$n =$ 0, 1, 2, 3, 4, 5

Should we write an explicit or recursive definition? Why?

What is the definition?  

\[ a_n = (n + 1)^2 \text{ explicit} \]
5. $a_n = \frac{n(n+1)}{2}$

Is this an explicit or recursive definition?

explicit

Write the first 6 terms of the sequence.

\[0, 1, 3, 6, 10, 15...\]

6. $a_0 = 1$

$a_1 = 3$

$a_{n, 2} = 2(a_{n, 1} + a_n)$

$n = 0$

$a_2 = 2(a_1 + a_0)$

$a_2 = 2(3 + 1)$

Is this an explicit or recursive definition?

recursive

Write the first 6 terms of the sequence.

$a_0, a_1, a_2, a_3, a_4, a_5$

\[1, 3, 8, 22, 60, 164...\]

$a_{n+2} = 2(a_{n+1} + a_n)$

$a_3 = 2(a_2 + a_1)$

$a_3 = 2(8 + 3) = 22$
7. \( \langle 5, 15, 45 \ldots \rangle \)

\[ n = 0 \quad 1 \quad 2 \]

what's the pattern?

\( x^3 \)

Should we write an explicit or recursive definition? *or maybe both ;)*

recursive definition: \( a_0 = 5 \)
\( a_{n+1} = a_n \cdot 3 \)

explicit definition: \( a_n = 5 \cdot 3^n \)
Sums of sequences

8. Arithmetic sequences... what are they?

\[ <1, 4, 7, 10... > \] each time \[ a_0 = 1 \]
\[ k = 3 \]

\[ <0, 5, 10, 15, 20... > \] each time \[ a_0 = 0 \]
\[ k = 5 \]

\[ <1, 2, 3, 4... > \] each time \[ a_0 = 1 \]
\[ k = 1 \]

9. \[ <1, 4, 7, 10... > \]

What's the recursive definition?

\[ a_0 = 1 \]
\[ a_{n+1} = a_n + 3 \]
10. Find the sum, $S_n$, of the first $n$ sequences in terms of $a_0$ and $k$. $S_n = a_0 + a_1 + a_2 + \ldots + a_{n-1}$

$n=2 \rightarrow S_2 = a_0 + a_1 = a_0 + (a_0 + 3) = a_0 + (a_0 + k)$
$S_2 = 2a_0 + k$

$n=3 \rightarrow S_3 = a_0 + a_1 + a_2 = a_0 + (a_0 + k) + (a_0 + 2k)$
$S_3 = 3a_0 + 3k$

$n=4 \rightarrow S_4 = a_0 + a_1 + a_2 + a_3 = a_0 + (a_0 + k) + (a_0 + 2k) + (a_0 + 3k)$
$S_4 = 4a_0 + 6k$

$n=5 \rightarrow S_5 = a_0 + a_1 + a_2 + a_3 + a_4 = a_0 + (a_0 + k) + (a_0 + 2k) + (a_0 + 3k) + (a_0 + 4k)$
$S_5 = 5a_0 + 10k$

11. How can we generalize $S_n$ for an arithmetic series?