

Week 4, Winter Quarter

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1 Problems

1. (BAMO 2015)(20min + 5min) In a quadrilateral the two segments connecting the midpoints of its opposite sides are equal in length. Prove that the diagonals of the quadrilateral are perpendicular.
2. (25min+10min)(BAMO 2007) Let N be the number of ordered pairs (x, y) of integers so that

$$x^2 + xy + y^2 \leq 2007.$$

Remember, integers may be positive, negative, or zero!

- a) Prove that N is odd.
 - b) Prove that N is not divisible by 3.
3. (10min +5min)Let $n \in \mathbb{N}$ be a positive integer. Let $\tau(n)$ be the sum of divisors of n . Find a description of this function in terms of prime factorization of n .
 4. (15min+10min)(1981 Kurschak Competition) For a positive integer n , let $r(n)$ denote the sum of the remainders when n is divided by $1, 2, \dots, n$ respectively. Prove that there are infinitely many n so that $r(n) = r(n + 1)$.