

PARITY

MATH CIRCLE (BEGINNERS) 10/30/2011

Parity is the property of an integer to be even (i.e. divisible by 2) or be odd (i.e. not divisible by 2). Despite the simplicity of this concept, parity can be used in showing that many arrangements are impossible to make. For example, it is impossible to find 5 odd numbers that add up to 100, because the sum of every 5 odd numbers is an odd number, whereas 100 is even. Below, you will find more problems that can be solved using parity.

In order to prove general statements about odd and even numbers, the following fact is useful: An even number is one which can be written as $2k$ for some integer k . For instance, $10 = 2(5)$. Similarly, an odd number is one which can be written as $2m+1$ for some integer m . For example, $7 = 2(3) + 1$. If we need to write down a generic even or odd number, we can just use those expressions. If perhaps we wanted to write down two possibly different even numbers, we could write $2m$ and $2n$ (or $2a$, $2t$, $2x$, etc.).

1) Prove that the sum of two odd numbers is even and the product of two odd numbers is odd.

2) Show that if the sum of two integers is odd then their product is even. (Hint: What are the possibilities for the parity of the two integers?)

3) Janet and her friends stand in a circle. It turns out that both neighbors of each child are of the same gender. If there are five boys in the circle, how many girls are there?

4) Can one make change of a 25-ruble bill, using in all ten bills each having a value of 1, 3, or 5 rubles?

5) Given two integers a and b , consider the expression $(a - b) \cdot a \cdot b$. Can you determine if it is even or odd?

6) On a chessboard, a knight starts from square $a1$, and returns there after making several moves. Explain why the knight must have made an even number of moves.

7) Can a 5×5 squares checkerboard be covered by 1×2 dominoes?

8) There are 100 soldiers in a detachment, and every evening three of them are on duty. Can it happen that after a certain period of time each soldier has shared duty with every other soldier exactly once?

9) Seven gears are placed on a plane, arranged in a circular chain. Can all the gears rotate simultaneously?

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg “Mathematical Circles (Russian Experience)”
- Previous UCLA Math Circle notes