

# Lesson 4: Invariants and Geometric Constructions

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## 1 From Last Week

### Problem 3.

Consider the numbers 1 through 6 placed on a circle in order. It is allowed

- a) to subtract one from three numbers no two of which are adjacent;
- b) to add one to three consecutive numbers;
- c) to do both.

Is it possible to use these operations to make all six numbers equal?

### Problem 5.

Let  $P$  be a point outside of a given circle. Show how to construct the two tangent lines from  $P$  to the circle.

## 2 New Problems

### Problem 1.

Consider a  $3 \times 3$  square with the lower-left corner colored black, and the rest colored white. Is it possible to make all squares white by repeatedly changing all the colors in a row or column?

### Problem 2.

In the alphabet of the “Mumbo-Yumbo” tribe there are only two letters – M and U. Two words have the same meaning if and only if one of them can be transformed into the other by inserting or removing the strings “MU” and “UUMM” at arbitrary places in the word arbitrarily many times.

- a) Do the words “UMM” and “MUU” have the same meaning?
- b) Do the words “UUUMMM” and “UUUUMMMM” have the same meaning?

### Problem 3.

Given two segments  $a$  and  $b$ , construct a right triangle  $ABC$  with hypotenuse  $BC$  equal to  $a$  and leg  $AC$  equal to  $b$ . For which values of  $a$  and  $b$  is it possible?

### Problem 4.

Given two segments  $a$  and  $b$  and angle  $\alpha$ , construct a triangle  $ABC$  with angle  $A$  equal to  $\alpha$ , side  $BC$  equal to  $a$  and side  $AC$  equal to  $b$ .