

Lesson 3: Weighings, Invariants and Geometric Constructions

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Problem 1.

You are given four coins with weights $1g$, $2g$, $3g$ and $4g$, but you do not know which ones are which. Is it possible to determine the weight of each coin using a balance scale at most 4 times?

Problem 2.

All four corners of an 8×8 board are colored black, and the rest of the squares are white. You are allowed to change the colors of all squares in a single row or column. Show that it is impossible to use such operations to make all the squares white.

Problem 3.

Consider the numbers 1 through 6 placed on a circle in order. It is allowed to add one to three consecutive numbers, or to subtract one from three numbers no two of which are adjacent. Is it possible to use these operations to make all six numbers equal?

Problem 4.

Show how to construct the center of a given circle.

Problem 5.

Let P be a point outside of a given circle. Show how to construct the two tangent lines from P to the circle.