

## Multiplying Negative Numbers

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### Warm-Up

An elevator takes 7 seconds to go from the first floor to the third floor. How long will it take to go from the first to the ninth floor?

*7 seconds  $\times$  4 = 28 seconds*

We're all familiar with *multiplying two positive numbers*. Let's do a couple of examples.

***Problem 1:*** Solve the following.

a.  $4 \times 2 = 8$

b.  $3 \times 3 = 9$

c.  $10 \times 5 = 50$

When multiplying **two positive numbers**, will your answer be negative or positive?

*Positive*

What if we were multiplying *one positive and one negative number*?

For example, what is  $4 \times (-2)$ ? To understand what the answer would be, we're going to approach this in three ways:

***Problem 2:*** What is  $4 \times (-2)$ ?

a. What is  $4 \times 0 = 0$

b. What is  $2 + -2 = 0$  (Use a number line, if necessary)

c. Since we know that  $2 + -2 = 0$ , let's replace the 0 in  $4 \times 0$  with the following:

i.  $4 \times 0 = 0$

ii.  $4 \times (2 + -2) = 0$

d. Now, if we distribute the 4, what do we get?

i.  $(4 \times 2) + (4 \times -2) = 0$

e. What is  $4 \times 2$ ? **8**

f. Let's replace  $4 \times 2$  with what we found in part (e).

i.  $(8) + (4 \times -2) = 0$

g. What does this tell you about  $4 \times -2$ ?

i.  $4 \times -2 = -8$

When multiplying **one positive and one negative number**, will your answer be negative or positive?

Negative

**Problem 3:** Also, remember that *multiplication* is *repeated addition*. Let's use this idea to see what happens when we multiply **one negative and one positive number**.

a. Expand the following multiplication problems in terms of addition.

i.  $4 \times 2 = 4 + 4 = 8$

ii.  $3 \times 3 = 3 + 3 + 3 = 9$

iii.  $(-3) \times 3 = -3 + -3 + -3 = -9$

iv.  $(-10) \times 4 = -10 + -10 + -10 + -10 = -40$

When multiplying **one negative and one positive number**, will your answer be negative or positive?

Negative

## Red Chilli Pepper Problem

*A group of 15 children gathered 100 mushrooms. Prove that at least two of them must have gathered the same number of mushrooms.*

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + a + b = 100.$$

$a + b$  in blue must add up to 9. No matter what, we'll end up using at least one number that we have previously used. Thus, at least two of them must have gathered the same number of mushrooms.

$$\text{Ex. } 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 9 = 100$$

**Problem 4:** What if **both** of our numbers **were negative**? We'll use a similar approach in *Problem 2*. Suppose we are trying to find what  $(-4) \times (-2)$  equals.

h. What is  $-4 \times 0 = 0$

i. What is  $2 + -2 = 0$  (Use a number line, if necessary)

j. Since we know that  $2 + -2 = 0$ , let's replace the 0 in  $-4 \times 0$  with the following:

i.  $-4 \times 0 = 0$

ii.  $-4 \times (2 + -2) = 0$

k. Now, if we distribute the -4, what do we get?

i.  $(-4 \times 2) + (-4 \times -2) = 0$

l. What is  $-4 \times 2$ ?

$-8$

m. Let's replace  $-4 \times 2$  with what we found for part (l).

i.  $(-8) + (-4 \times -2) = 0$

n. What does this tell you about  $-4 \times -2$ ?

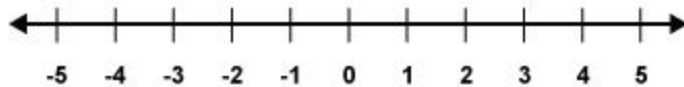
i.  $-4 \times -2 = 8$

When multiplying **two negative numbers**, will your answer be negative or positive?

Positive

**Problem 5:** We can also think of multiplication by negative numbers as a direction switch.

- a. A number tells us two things:
  - i. The distance from **0** on the number line
  - ii. The **direction** in which to travel this distance.



- b. Then multiplying a number by -1 doesn't change the **distance**, but flips the **direction** to the other side.
- c. Using the number line above, let's calculate:
  - i.  $(-1) \times 4 = -4$
  - ii.  $(-1) \times (-4) = 4$
- d. Using this idea,  $(-4) \times (-2) = (-1) \times 4 \times (-1) \times 2 = (-1) \times (-1) \times 4 \times 2 = 8$ 
  - i. *What happens to the direction when we multiply  $(-1) \times (-1)$ ?  
It changes the direction twice.*

**Problem 6:** Compute:

- a.  $4 \times 2 \times (-3) = -24$
- b.  $(-5) \times 4 \times (-5) = 100$
- c.  $(-3) \times (-8) \times (-2) = -48$
- d.  $(-2) \times (-3) \times (-4) \times (-5) = 120$

e. *What happens when you multiply 3 negative numbers together?*

The answer is negative.

f. *How about 4 negative numbers?*

The answer is positive.

g. *Do you see a pattern? If yes, please describe and explain.*

If we multiply an even number of negative numbers, the answer is positive.

If we multiply an odd number of negative numbers, the answer is negative.