Multiplying Negative Numbers

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Warm Up

An elevator takes 7 seconds to go from the first floor to the third floor. How long will it take to go from the first to the ninth floor?

We're all familiar with ***multiplying two positive numbers***. Let's do a couple of examples.

Problem 1: Solve the following.

a. 4 x 2 =

b. 3 x 3 =

c. 10 x 5 =

When multiplying **two positive numbers**, will your answer be negative or positive?
_____

What if we were multiplying ***one positive and one negative number***?

For example, what is *4 x (-2)?* To understand what the answer would be, we're going to approach this in three ways:

Problem 2: What is 4 x (-2)?

a. What is 4 x 0 = _____

b. What is 2 + -2 = _____ (Use a number line, if necessary)

c. Since we know that 2 + -2 = _____, let's replace the 0 in 4 x 0 with the following:

i. 4 x 0 = _____

ii. 4 x (___ + ____) = _____

d. Now, if we distribute the 4, what do we get?

i. (4 x ___) + (4 x ___) = _____

e. What is 4 x 2? _____

f. Let's replace 4x2 with what we found in part (e).

i. (____) + (4 x __) = _____

g. What does this tell you about 4 x -2?

i. 4 x -2 = _____

*When multiplying **one positive and one negative number**, will your answer be <u>negative</u> or <u>positive</u>?*

_____

<u>Problem 3:</u> Also, remember that *multiplication* is *repeated addition*. Let's use this idea to see what happens when we multiply ***one negative and one positive number***.

a. Expand the following multiplication problems in terms of addition.

i. 4 x 2 = ____ + _____ =

ii. 3 x 3 = _____ + _____ + _____ =

iii. (-3) x 3 = _____ + _____ + _____ =

iv. (-10) x 4 = _____ + _____ + _____ + _____ =

*When multiplying **one negative and one positive number**, will your answer be <u>negative</u> or <u>positive</u>?*

_____

Red Chilli Pepper Problem

A group of 15 children gathered 100 mushrooms. Prove that at least two of them must have gathered the same number of mushrooms.

<u>Problem 4:</u> What if **both** of our numbers **were negative**? We'll use a similar approach in *Problem 2*. Suppose we are trying to find what ***(-4) x (-2)*** equals.

h. What is -4 x 0 = _____

i. What is 2 + -2 = _____ (Use a number line, if necessary)

j. Since we know that 2 + -2 = _____, let's replace the 0 in -4 x 0 with the following:

   i. -4 x 0 = _____

   ii. -4 x (___ + ____) = _____

k. Now, if we distribute the -4, what do we get?

   i. (-4 x ___) + (-4 x ___) = _____

l. What is -4 x 2?

   _____

m. Let's replace -4x2 with what we found for part (l).

   i. (_____) + (-4 x __) = _____

n. What does this tell you about -4 x -2?

i. -4 x -2 = _____

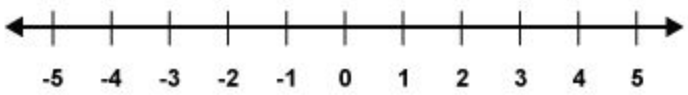
*When multiplying **two negative numbers**, will your answer be <u>negative</u> or <u>positive</u>?*   _____

<u>Problem 5:</u> We can also think of multiplication by negative numbers as a direction switch.

a. A number tells us two things:

i. The distance from _____ on the number line

ii. The _____ in which to travel this distance.



b. Then multiplying a number by -1 doesn't change the _____, but flips the _____ to the other side.

c. Using the number line above, let's calculate:

i. (-1) x 4 = _____

ii. (-1) x (-4) = _____

d. Using this idea, (-4) x (-2) = (-1) x ____ x (-1) x ____ = (___) x (___) x 4 x 2 = 8

i. What happens to the direction when we multiply (-1) x (-1)?

<u>Problem 6:</u> Compute:

a. 4 x 2 x (-3) =

b. (-5) x 4 x (-5) =

c. (-3) x (-8) x (-2) =

d. (-2) x (-3) x (-4) x (-5) =

e. What happens when you multiply 3 negative numbers together?

f. How about 4 negative numbers?

g. Do you see a pattern? If yes, please describe and explain.