

Lesson 4: Algebra and remainders.

Konstantin Miagkov

Problem 1.

- a) The straight line $y = 7x/15 + 1/3$ passes through two integral points: $(10, 5)$ and $(-20, -9)$. Does it pass through any other integral points?
- b) The graph of a function $y = kx + b$ passes through two distinct integral points. Are there any other integral points on this graph?
- c) Does there exist a linear function $y = kx + b$ such that its graph passes through exactly one integral point?

Problem 2.

Solve the equation:

$$\begin{cases} \frac{x}{x+1} + y^2 = 4 \\ y^2 - \frac{5x}{x+1} = -14 \end{cases}$$

Problem 3.

- a) Let a, b be positive integers. Show that there exist unique nonnegative integers q, r such that $a = bq + r$ and $r < b$.
- b) Let a, b be integers. Show that there exist unique integers q, r such that $a = bq + r$ and $0 \leq r < |b|$.

Problem 4.

Show that $n^5 + 4n$ is divisible by 5 for any integer n .

Problem 5.

Let x, y, z be integers such that $x^2 + y^2 = z^2$. Show that at least one of x, y, z is divisible by 3.

Problem 6.

Is it possible to write 1986 as a sum of 6 squares of odd numbers?